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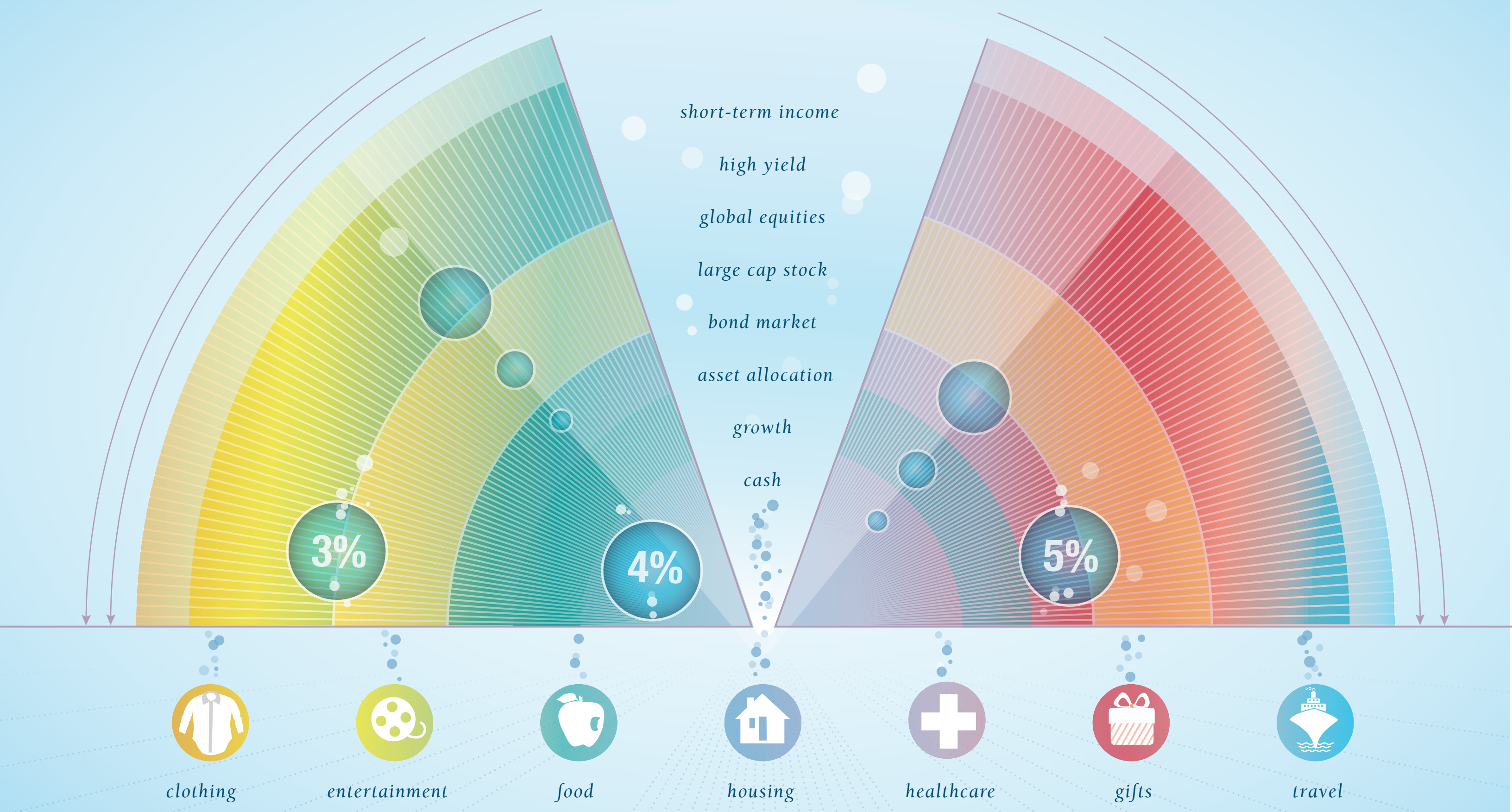
From Savings to Income: **Retirement Drawdown Strategies**



RESEARCH STUDY

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From Savings to Income: Retirement Drawdown Strategies

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01 Introduction

More than one in three Americans is aged between 45 and 74 and is either in retirement or thinking about it.[i] For most of these 100 million people, the days of pension plans are all but gone. Less than one in five workers is covered by a defined benefit retirement plan.[ii] Meanwhile, the 401(k) experiment is well underway. These defined contribution plans make individuals responsible for financing and managing their own retirements.

Given the massive cohort involved, informed decisions about retirement financing will be essential not only to retirees, but to the health of the U.S. economy. This study is intended to determine whether there is a retirement drawdown strategy that strikes a balance between prematurely exhausting savings and under-spending during retirement. It is intended to help those nearing retirement consider how to effectively draw income from their savings after they have left the workforce.

Beyond the 4 Percent Rule

In 1994, William Bengen developed the 4 Percent Rule for drawing down assets in retirement: Calculate 4 percent of the nest egg in the first year of retirement, and live off that amount for every subsequent year (adjusting for inflation). Bengen found that this strategy, based on simulated historical portfolios that had invested 50/50 in stocks and bonds, would allow retirement savings to last at least 30 years.

The 4 Percent Rule continues to be a benchmark. For a rough estimate of how much is needed for retirement, it's not bad. But no simple financial rule can take into account the complexity of real life.

This study moves beyond the 4 Percent Rule and examines an array of retirement drawdown strategies. These strategies are meant to respond to the range of financial risks and opportunities that retirees may face.

The analysis focuses strictly on strategies for drawing down retirement savings. It is not an asset allocation study. A 50/50 stock and bond allocation is assumed as a framework for the analysis. Asset allocation and annuity income will be the subject of future AIER research.

Our analysis offers some guiding principles that retirees can use when building a retirement drawdown strategy.

There is no hard and fast rule. No obvious winning strategy can be found across the spectrum of historical market returns and inflation. The success of a retirement drawdown strategy is dependent on investment returns that have been historically erratic. Randomness and timing play a huge role in determining which strategy offers the best simulated outcome. Table 1 illustrates selected historical retirement dates and how the best sustainable rate of withdrawal fluctuates depending on indeterminate market conditions.

Early retirement behavior matters most. The first 5-10 years of retirement have an outsized impact on long-term success. Spending less during these years provides a better likelihood of positive outcomes over the retirement horizon.

A flexible drawdown strategy leads to better retirement outcomes. The best simulated results come from strategies that increase the drawdown percentage later in retirement.

A prudent starting point is 3-5 percent. Initial drawdowns below 3 percent should be considered only for the most risk-averse retirees or when future returns are expected to be well below the historical average. Initial drawdowns above 5 percent should be considered only if risk tolerance is high and the strategy chosen responds to market drops.

Retirement planning research is often directed toward financial planners instead of retirees. AIER assumes that retirees themselves are most well-suited to determine personal retirement strategies and emphasizes personal financial responsibility. This paper equips people with the information needed to begin thinking about that process.

TABLE 1. Best Drawdown Percentages Vary

Retirement Date	Optimal Drawdown Percentage for Constant Dollar Strategy
September 1929	4.0%
July 1930	5.5%
February 1932	7.0%
July 1933	5.0%
September 1936	4.0%
August 1948	7.0%
June 1954	4.5%
June 1961	3.5%
May 1965	3.0%
January 1974	4.5%

Notes and sources: Author's calculations. See appendices for model details.

02 Establishing Retirement Rules

The retirement nest egg has two distinct life phases: accumulation and decumulation. During the accumulation phase, the nest egg hopefully grows at a robust pace. Job earnings can soften the pain of market fluctuations that may cause savings to shrink. As long as there are more years of working income that can be saved, the nest egg should continue to grow through volatile periods. The decumulation phase is the period when savings are the primary source of household income. In that phase, when market fluctuations reduce the nest egg, there is no working income to replenish it.

There is ample information about how and where to invest assets during accumulation. Financial professionals are eager to service retirement assets during this phase. During retirement, strategic spending from savings is as important as the pre-retirement strategy. However, information about post-retirement spending strategies is less plentiful.

As noted, the field of retirement finance started in earnest when William Bengen established the 4 Percent Rule in 1994.[iii] The 4 Percent Rule dictates that a retiree calculate 4 percent of her nest egg in the first year of retirement and draw down that amount every subsequent year, adjusting for inflation. For example, a retiree with $500,000 in retirement savings would draw $20,000 every year in retirement, adjusting for inflation. Bengen's seminal work has inspired new research, much of it dedicated to debunking the rule.

Is the 4 Percent Rule Too Conservative?

Retirement success is often determined by whether assets are exhausted at the end of a simulated retirement. A strategy is deemed successful when a retiree doesn't outlive her money. An influential study from 1998 looked at success rates and found that a 4 percent drawdown had a 95 percent success rate for retirements of 30 years that had a 50/50 stock and bond allocation.[iv] This study suggested that "for stock-dominated portfolios, withdrawal rates of 3 percent and 4 percent represent exceedingly conservative behavior." This was the first of many studies that deemed the 4 Percent Rule too conservative, suggesting that retirees could be spending more.

A subsequent study from 2004 proposed "a maximum safe initial withdrawal rate as high as 5.8 percent to 6.2 percent depending on the percentage of the portfolio that is allocated to equities."[v]

Yet another study suggested that "risk-tolerant clients can maximize their [retirement outcomes] with a 7 percent withdrawal rate and a 70 percent stock allocation."[vi] This aligns with a 2001 study that found a 68 percent success rate over 30-year retirements with a 6 percent withdrawal rate and 75 percent stock allocation.

When measuring success as "not exhausting assets," all successes and all failures are treated equally. Actual shortfall or excess amounts are not taken into account. David Loeper states that "when 830 of 1,000 simulations meet all of the client's life goals and leave an estate value larger than the desired legacy goal, [it] is not an 83 percent chance of success. It is an 83 percent chance of excess."[vii] Loeper's report focuses on maximizing average lifetime consumption, arguing that the value of solvency is reduced when the risk of under-spending is taken into consideration. He writes that "few clients would view a plan as 'successful' for advice that caused them to sacrifice their only life to die on a death bed stuffed with money they wish they had spent."

Several new studies account for the amount of excess or shortfall. One study measured expected retirement lengths and pension income and concluded that a 4 percent spending rule is optimal only in very limited and unrealistic circumstances.[viii] Another study compared retirement outcomes that consider under-spending as impairment to lifetime utility.[ix] In other words, it assumes that individuals look most favorably on a retirement where spending is maximized and savings are exhausted exactly at the end of retirement. The work concludes that the 4 Percent Rule is too conservative in most real-life circumstances.

These studies represent the idea that financial planning is too variable and complex to be summed up in a rule. They generally focus on maximizing lifetime spending rather than minimizing the risk of exhausting assets, suggesting the 4 Percent Rule is too conservative for the average household.

Is the 4 Percent Rule Too Aggressive?

It is also argued that the 4 Percent Rule is too aggressive. This case is built on the notion that historical rates of return from stocks and bonds, upon which many studies are built, likely overstate future returns.

Dr. Wade Pfau, a retirement economics scholar, observed historical returns in global equity markets and found that the United States has experienced asset returns from stocks and bonds over the last century that are higher than those in other developed economies.[x] He notes that when looking toward the future, it is "not clear whether asset returns in the 21st century will continue to be as great as in the 20th century, or whether savers and retirees should plan for something closer to the average international experience."

Pfau attempts to estimate sustainable withdrawal rates based on predicted rather than historical returns. He suggests that high current stock valuations and low dividend yields will likely be followed by lower future returns, especially in the next 5-10 years. These lower returns will mandate lower sustainable withdrawal rates than history suggests. He estimates that sustainable withdrawal rates have fallen below 3 percent since 1999 and maybe below 2 percent since 2003.

The first 10 years of retirement are the most critical. During the drawdown phase, an initial 5-10 years of poor performance can lead to poor outcomes that cannot be remedied during subsequent retirement years when there is no income to replenish the nest egg. This is known as sequence risk: the risk that returns in the early years of retirement won't produce adequate growth to allow the nest egg to last.

Another study estimated return expectations based on current, rather than historic, market conditions.[xi] It described a scenario in which the 4 Percent Rule results in a failure rate of up to 57 percent, demonstrating that a 4 percent drawdown is much riskier than historical returns suggest. Even under an assumption that historical average returns reappear in five years, failure rates range from 18-22 percent, which affirms the dangers of sequence risk and poor returns early during retirement.

Pfau theorizes an optimal retirement

asset allocation scenario that starts conservatively and increases in riskiness with age.[xii] This suggests that a low equity allocation is most prudent during the early years of retirement but less critical late in retirement, which challenges conventional wisdom. His analyses use market returns that are lower than historical averages and predict much lower success rates than those based on historical returns.

The literature suggests that the 4 Percent Rule cannot meet the needs of all retirees. Forecasting returns and predicting that they will be lower than historical data indicates that these studies are cautious. They propose that lower drawdown rates must be considered given the uniquely high return environment encountered in the United States during the last century. They focus on reducing risk rather than maximizing lifetime spending.

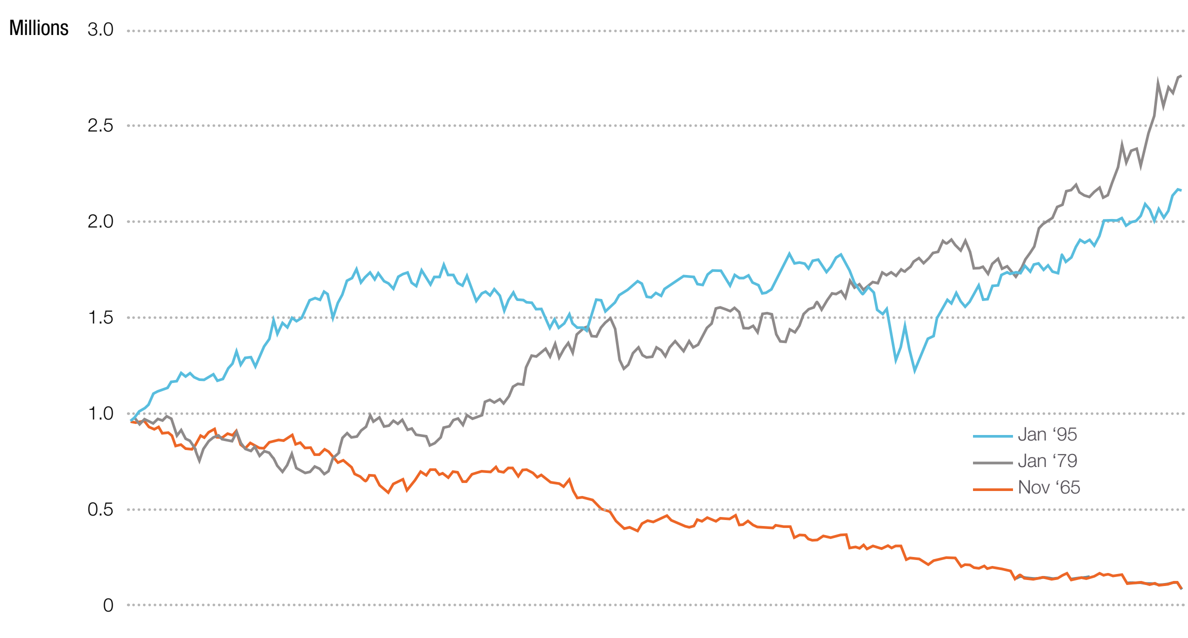
Reviewing the 4 Percent Rule with Recent Data

Since 1994, there have been 19 additional years of return data to analyze the successes of those who have followed the 4 Percent Rule. If a retiree followed the 4 Percent Rule and invested 50/50 in stocks and bonds, how would her nest egg look today?

To examine this question, AIER constructed simulated investment periods to mimic actual retirement periods. It's important to remember that any analysis based on historical returns relies on a limited amount of data. There have been less than three unique and independent 30-year return windows since our investment returns data begin in 1928 (1928-1957, 1958-1987, 1988-2017).

Overlapping data series are required to obtain a meaningful number of simulated retirements. We simulated 35-year retirement periods beginning in January 1928. The first period includes returns from January 1928 through December 1962. The second period includes returns from February 1928 through January 1963, and so on. This approach allows for 613 separate, 35-year retirement periods, the

FIGURE 1. 19 Year Retirement Balances
$1 MILLION INITIAL BALANCE: 50/50 STOCKS/BONDS, 4% INFLATION ADJUSTED DRAWDOWN



Source: Author's calculations based on Bloomberg, Morningstar, and Ibbotson data.

The Importance of Rebalancing

We have assumed in the simulations (Table 2) that portfolios rebalance once a year to a 50/50 stock/bond allocation (see Appendix B for full methodology). Markets have a tendency toward mean reversion. For example, the simulated portfolio that started in January 2008 would be worth only about $820,000 by the end of 2008. By rebalancing the portfolio and moving funds from bonds to stocks, the simulated portfolio was able to reap the benefits of the higher equity returns during 2009 and would return to a value of about $900,000 by the end of 2009. With less than 3 percent total inflation over these two years, this outcome is not that bad, as a result of timely rebalancing. The first year of retirement would have certainly been challenging, and moving assets into equities at the end of 2008 would have been very difficult to stomach, but the outcome speaks for itself.

last of which spans from January 1979 through December 2013. This number of simulations offers a wide range of outcomes. The following comparisons show outcomes for 19 years into retirement.

People who have retired since 1995 have done well compared to retirees from earlier periods. Had a worker retired on January 1, 1995 and followed the 4 Percent Rule, today she would have the inflation-adjusted equivalent of about 2.16 times her original nest egg. There are only 23 historical simulations that would have exhibited a better outcome 19 years into retirement.

Figure 1 shows simulated results for the cohort retiring in January 1995 compared with the best and worst-case 19-year retirements in the sample. All dollar figures are inflation-adjusted. While the 4 Percent Rule has resulted in an array of 19-year outcomes, the January 1995 retiree would be better off only if she had retired in late 1978, early 1979, 1948 or 1949.

Do these results indicate that the 4 Percent Rule is a reasonable guide? A couple that retired in 1995 at age 65 would be 84 years old today. If they started with $1 million and followed the 4 Percent Rule, they would have about $2.16 million today, after adjusting for inflation. The couple who started drawing $40,000 in 1995 would currently be drawing about $62,000 annually. They have clearly not maximized potential spending; they could have been drawing a higher annual income and still enjoyed a safe margin of error. On the other hand, the couple is now in a financially comfortable position. They still have a nest egg and can easily endure unexpected costs or market downturns. They can choose to live it up during the rest of their lives, and potentially leave a large bequest to their heirs.

Economists reason that this couple could have increased lifetime utility, or happiness, by spending more during the last nineteen years. This suggests that 4 Percent Rule payouts are too low. But if 4 percent per year provided a comfortable living, few investors would complain about this outcome.

The recent experience of several cohorts that retired throughout the 2000s is represented in Table 2. These individuals were retiring throughout a bear market, the "lost decade" of 2001-2010. Yet the table shows that these outcomes, based on the 4 Percent Rule and covering time spans ranging from 5-19 years, would have exceeded the majority of historical outcomes generated by corresponding time spans.

Only the retiree class from January 2000 was outperformed by the majority of its comparable historical time spans.

TABLE 2. 4 Percent Rule Simulations

Retirement Date	Number of Years into Retirement	Ending Nominal Value (Dec. 2013)	Ending Inflation Adjusted Value (Dec. 2013)	Percentile Historical Simulations
Jan 1995	19	3,360,752	2,158,678	96%
Jan 2000	14	1,142,034	824,635	49%
Jan 2002	12	1,354,988	1,027,202	59%
Jan 2003	11	1,540,512	1,195,873	65%
Jan 2004	10	1,300,051	1,028,057	59%
Jan 2005	9	1,219,823	996,134	58%
Jan 2006	8	1,218,284	1,028,559	59%
Jan 2007	7	1,147,035	993,296	58%
Jan 2008	6	1,165,075	1,050,027	60%
Jan 2009	5	1,486,912	1,341,308	72%

Initial Balance: $1 million
50/50 stocks/bonds
$40,000 Annual Drawdown, adjusted for inflation
Source: Author's calculations. Returns data from Bloomberg, Morningstar, and Ibbotson.

In that case, 51 percent of historical 14-year spans had higher inflation-adjusted ending values. But even that retiree is in a relatively strong financial position, as she has more nominal wealth than she had at retirement 14 years ago, and she would have experienced relatively modest inflation. This retiree would be drawing about $55,000 this year to match her inflation-adjusted $40,000 original withdrawal.

In light of the lost decade, these positive relative outcomes for recent retirees may appear surprising. It is important to remember that the stock market hit bottom in March 2009, and the current bull market is among the longest and strongest on record.

Back in 2010, recent retirees faced a bleaker outlook. A 2010 study found that retirees from the year 2000 were on pace to experience the worst retirement outcomes since the mid-1920s.[xiii] This cohort of retirees was in the bottom 2-5 percent of historical retirement outcomes by nominal wealth in 2010. (Mild inflation during the decade suggests that this cohort was at about the 15th-20th percentile of inflation-adjusted retirement outcomes.) The situation looked bad, but the three subsequent years of strong equity returns may have brightened the outlook for these retirees.

The 4 Percent Rule may be too simplistic to capture the nuances of retirement financing, but as the above examples illustrate, it can be a helpful starting point toward gaining perspective on the topic of retirement savings drawdown.

03 Retirement Drawdown Strategies for the 21st Century

The analysis that follows is based on several alternative retirement drawdown strategies. The primary criterion for a strategy to be included in this analysis was that it could be easily replicated by the average retiree. This excludes some creative strategies that perform well in simulations but are too difficult for the average investor to incorporate without a financial planner. Strategies that include insurance product annuities are also excluded. Annuities will be researched in future efforts. For a full description of strategies considered in this analysis, please see the box on the following page.*

Eight strategies for drawing retirement income are considered here. They fall into three broad categories: *constant dollar, constant percentage,* and *increasing percentage.* The *constant dollar* approach selects an annual drawdown amount and adjusts for inflation. This is the strategy employed under the 4 Percent Rule. This strategy does not adapt to market returns. It does adjust to inflation, which allows for an unwavering stream of income throughout retirement.

The *constant percentage* approach withdraws a fixed percentage of the portfolio annually. This strategy allows for more adaptability to market movements and is less likely to exhaust assets, because when the nest egg decreases in value, the drawdown amount decreases similarly. This strategy is subject to greater yearly income variability. When market returns are poor, the *constant percentage* strategy responds by drawing a reduced income.

The *increasing percentage* approach draws an initial percentage of the portfolio and adjusts that percentage upward throughout retirement. As the retiree ages, a higher percentage drawdown is more acceptable, because the portfolio needs to last a shorter period. This strategy is adaptable to market conditions but can be susceptible to drawdown variability.

Throughout the following analysis, the merits of these strategies are considered under various scenarios. The "utility model," a way to measure the desirability of an outcome, is described in Appendix A. The model construction methodology can be found in Appendix B. A quantitative comparison of the strategies can be found in Appendix C. Monte Carlo simulation results can be found in Appendix D.

Strategies that are untested due to their complexity include the *constant failure percentage* methodology and the *mortality updating failure percentage.*[xiv] These strategies adjust drawdown percentages upward as retirees get older, and they are most similar to the increasing percentage strategies listed in drawdown strategies 6-8. Another

* Tax-advantaged retirement vehicles, such as 401(k) plans, mandate a certain level of distribution after age 70. This is known as the required minimum distribution. Our model uses drawdown levels that are less than the required minimum distribution. In these cases, we assume that the retiree would be forced to withdraw the money but not to spend the money. In other words, the money that is withdrawn can be reinvested in a different vehicle.

The 8 Drawdown Strategies:

1 Constant dollar, inflation adjusted: This strategy starts with a constant dollar amount and adjusts it annually for inflation. This strategy is not adaptable to market returns. Rather, it aims for a sustainable drawdown level that can leave the retiree with an unwavering stream of income throughout retirement. This is the strategy employed under the 4 Percent Rule.

2 Constant percentage: This strategy withdraws a constant percentage of the portfolio at the beginning of every year. It allows for more adaptability to market movements and is less likely to exhaust assets. When the portfolio decreases in value, the drawdown amount decreases similarly. This strategy suffers from a higher rate of drawdown variability.

3 Smoothed percentage: This strategy is similar to the constant percentage strategy, except it uses the average of the previous three years of drawdowns to smooth consumption. If the previous three years' drawdown averaged $40,000 and the constant percentage amount would be $50,000, the smoothed amount would $45,000.

4 Constant-percentage ceiling: This strategy is similar to the constant percentage strategy, except it caps the drawdown in any given year to the original drawdown amount, adjusted for inflation. Strong financial returns are not met with increased drawdown amounts under this strategy.

5 Constant-percentage floor: This strategy is the same as the constant percentage ceiling strategy, but a floor is implemented instead of a ceiling. This strategy offers higher upside, but has the potential to exhaust savings when the floor is set too high.

6 Inflation-adjusted percentage: This strategy starts with a percentage to draw down and adjusts that percentage with inflation. This will adjust the percentage upward during times of positive inflation. The strategy assumes that as the retiree ages, a higher-percentage drawdown is more acceptable. This strategy is adaptable to market conditions, but can be susceptible to drawdown variability.

7 Increasing percentage: This strategy starts by drawing a certain percentage of the portfolio and increasing that percentage by 5 percent each year. For example, draw 3 percent the first year, 3.15 percent the second year, 3.3075 percent the third year, etc. The drawdown percentage is capped at 10 percent.

8 Required minimum distribution percentage (RMD): This strategy uses the IRS's required minimum distributions to calculate an annual drawdown percentage. The first five years of retirement use an inflation-adjusted constant-dollar strategy, and then the RMD assumptions are used from ages 70-plus.

common strategy is to spend only interest and dividends.[xv] This strategy was not tested because the portfolio allocation is not focused on dividend and interest income.

AIER's Model for Retirement Finance

As Mark Twain popularized, there are lies, damn lies, and then there are statistics. Economists and statisticians can pick and choose outcomes to make a story that fits a construct. The 4 Percent Rule can look wonderful or terrible depending on the data chosen, but the truth lies somewhere between. This research effort does not advocate for or against any single strategy, but illustrates how various strategies would have fared in a historical context. Understanding which strategies fared best under certain environments can help provide a construct to help shape personal decision-making.

For each drawdown strategy, we apply initial drawdown amounts ranging from 2 to 7.5 percent. Then, we examine the performance that would have resulted from actual market returns that occurred between 1928 and 2013. We seek to determine which combination of drawdown strategy and initial withdrawal amount is best under a spectrum of investment conditions.

Each of these simulated retirement combinations results in a stream of income over the retirement period, from which we calculate a *utility score.* The score reflects both average annual spending and the minimum annual spending. (See Appendix A for full calculation of the utility score.) We examined eight drawdown strategies and 12 different initial annual percentage withdrawals. The resulting 96 strategy/drawdown combinations achieve higher utility scores when they allow for high and stable annual spending. Utility scores are diminished when the portfolio is prematurely exhausted or when retirement spending is too low.

An *increasing percentage* strategy that uses a 2 percent initial drawdown, for example, will produce different annual retirement income in different markets. The utility model compares results for this strategy/drawdown combination across hundreds of different investment environments to calculate its utility score. These results are compared with utility scores calculated for the same strategy using a 3 percent drawdown amount, and so on. To further examine each combination, the analysis looks at worst-case scenarios, which combine longer-than-average retirement lengths with poor investment conditions.

The utility scores illustrate various tradeoffs. For instance, an increasing drawdown percentage starting at 4.5 percent offers the highest average utility. However, under the worst-case scenario, this strategy offers significantly lower utility than the same strategy that starts with a 3 percent drawdown.

Across all historical investment conditions, a 4.5 percent inflation-adjusted percentage drawdown results in a range of utility scores from 5.41 to 13.20.

Assuming a starting savings of $1 million, the utility score of 13.20 is derived from average annual retirement spending of $87,100 and minimum annual spending of $44,900. Such levels of spending can only be accomplished during periods of very high returns, as occurred during retirements that began in July 1932. The utility score of 5.41, however, results from average annual retirement spending of $33,900 and minimum annual spending of $20,200. These levels are lower as a result of poor returns during the retirement period that began in November 1965.

An average utility score is calculated by averaging the scores across each of the 553 historical rolling return periods between 1928 and 2013. As Table 3 shows, the average utility score for this particular strategy/drawdown was 8.19. During longer-than-average retirements with poor returns, however, the utility score was only 4.17.

This divergence in scores highlights a tradeoff that must be considered. Throughout our report, we provide average and worst-case utility scores for the various strategy/drawdown combinations.

Table 3 provides utility scores for several selected strategies and drawdown percentages. It shows the average scores and those in the worst-case scenario for each combination. How a person selects a strategy among these options depends on individual preference. Retirees that anticipate great longevity and are more pessimistic about future market returns should look closely at worst-case scores. Retirees that are more risk-tolerant may be more interested in average utility scores.

TABLE 3. Utility Scores for Six Selected Strategy/Drawdown Combinations

	Average Utility Score	Worst-Case Utility Score
Inflation-Adjusted Percentage		
4.5% drawdown	8.19	4.17
3.0% drawdown	7.28	6.13
Constant Dollar		
4.5% drawdown	7.93	2.65
3.5% drawdown	6.97	7.00
Constant Percentage		
7.0% drawdown	7.42	4.72
6.0% drawdown	7.33	4.76

Source: Author's calculations.

Historical vs. Monte Carlo Simulation

Our primary analysis uses historical returns data for simulation. Historical simulations use monthly returns as they occured. Results are corroborated by Monte Carlo simulation. The Monte Carlo simulations select months at random from January 1928 through December 2013 with replacement to create 1,000 unique return streams (a bootstrapping methodology).

Historical simulations are easy to interpret. They represent retirements that may have actually taken place beginning at different points in time. We prefer using the historical method as the primary analysis for its relative simplicity.

The Monte Carlo simulation is less obvious to interpret. The Monte Carlo simulation assumes that monthly returns are random and the future could comprise of any random selection of past monthly returns. Monte Carlo lacks some realism in that monthly asset return momentum is not considered. Monte Carlo simulation also assumes that monthly returns are independently distributed. Monte Carlo results tend to result in higher average utility scores. This is because strategies tend to fair best when returns are random and uncorrelated. Due to the randomness of return selection, a Monte Carlo simulation is unlikely to replicate, for instance, a stretch of high inflation such as that experienced during the 1970s. Likewise, it is unlikely that the simulations would replicate many negative returns months in a row, although these trends are prominent in the historical data. Nonetheless, the simulations offer a similar pattern to the historical simulations. Appendix D offers the full Monte Carlo results.

Differences from Previous Efforts

This analysis excludes certain components of other research, such as the level of risk aversion, a forgone consumption penalty, and age-adjusted preferences. This method is therefore simpler than other efforts on the topic, but it is sophisticated enough to provide meaningful direction.

Average annual spending, and therefore utility scores, may increase during periods of strong asset returns in our model. A strategy that increases spending during bull markets will achieve a higher utility score, which differs from other utility models that are based on a ratio of spending to maximum allowable spending.[xvi, xvii] AIER's model ranks two retirees that spend an inflation-adjusted $40,000 every year exactly equal regardless of the period in which they retired. Other utility models will often penalize one of these outcomes for not adapting to higher potential spending.

It is unclear whether the ratio of actual to maximum possible spending has any impact on happiness. Maximum possible spending could result in a very stressful end of retirement. Does ending retirement with exactly zero dollars maximize utility, as economists typically assume? There could be positive utility in leaving a bequest. There could be positive utility in maintaining a comfortable cushion throughout retirement.

Risk aversion is also common in utility functions. A risk-averse retiree may place a higher premium on the stability of income as opposed to the average spending level. A risk-tolerant, or risk-seeking, retiree may place a higher premium on average spending and accept more variability. The model shown here does not account for specific levels of risk aversion. A measure of risk aversion could be included, but it may not provide additional actionable information for the retiree. Studies have shown that actual risk aversion differs significantly from self-reported risk aversion. A table of outcomes based on levels of risk aversion may be of limited worth.

Finally, the utility function used here does not allow for preferences in near-term versus long-term spending. It is unclear whether it is preferable to have more spending today or to spend less today for the comfort of potentially higher spending tomorrow. Some utility measures provide a relative preference for near-term spending in retirement.[xviii] Although there may be a compelling argument in favor of a time preference, the preference for near-term spending has little impact in comparable studies and adds unnecessary complexity.

The best way to evaluate the utility measurement is to understand it contextually. A more risk-averse individual should think about maximizing utility scores under the worst-case scenario, whereas a risk-seeking retiree might think about maximizing utility scores under an average scenario.

04 Results

Highest Utility Scores

There is no clear winning strategy. Table 4 below provides utility measures based on selected strategies and drawdown percentages. It provides the measures where they are maximized on average and in the worst-case scenario.

As the table shows, drawdowns slightly more aggressive than 4 percent tend to achieve the highest average utility scores. An *inflation-adjusted percentage* (increasing with inflation) with a starting 4.5 percent drawdown provides the best average utility. When considering the Worst-Case scenario, a 3 percent *inflation-adjusted percentage* strategy is preferable, as noted in the previous section.

Initial drawdowns below 3 percent should be considered only for the most risk-averse retirees, or when return expectations are well below the historical average. Initial drawdowns above 5 percent should be considered only if a flexible strategy that responds to changing market returns is chosen and risk tolerance is high.

Average vs. Worst-Case Scenarios

For many historical retirement periods, the strategy with the highest average utility score was a *constant percentage "floor"* strategy (see box on page 10 for strategy definitions) that drew between 4.5 and 5.5 percent annually. This strategy is a hybrid between a *constant dollar* and *constant percentage* strategy. It sets a floor withdrawal but allows for increased spending when returns are above average. On the other hand, it can exhaust assets when the floor is set too high and returns are poor, similar to a *constant dollar* strategy.

TABLE 4. Utility Scores for Selected Strategies

	Inflation-Adjusted Percentage		Constant Dollar		Constant Percentage	
	4.5 percent drawdown	3.0 percent drawdown	4.5 percent drawdown	3.5 percent drawdown	7.0 percent drawdown	6.0 percent drawdown
Average Utility (Weighted Average Life Expectancy)	8.19	7.28	7.93	6.97	7.42	7.33
Long Retirement Average Utility	7.03	7.61	7.02	6.90	6.26	6.48
Poor Outcomes (5th Percentile Utility)	5.80	5.72	4.35	6.92	5.11	4.98
Worst-Case	4.17	6.13	2.65	7.00	4.72	4.76

This strategy would have fared well for all retirements from January 1930 through June 1935, lasting at least 40 years. This strategy would have exhausted assets, however, after only 23 years for retirements beginning in November or December 1936. It is a relatively high-risk strategy that would have happened to perform well on average during the historical simulations.

For longer retirements with poor outcomes (worst-case scenario), this strategy would have suffered. Utility scores during the worst-case scenario would have been about half that of an *inflation-adjusted percentage* strategy with a 3 or 3.5 percent starting drawdown. This exemplifies the tradeoff between these strategies. Where a *constant dollar* (or *constant percentage with floor*) strategy can excel on average, it can also suffer at the extremes.

Figure 2 compares the utility scores of a *constant dollar* strategy with the utility scores of an *inflation-adjusted percentage* strategy. The results show that the *constant dollar* strategy compares favorably with the *inflation-adjusted percentage* strategy during an average retirement. At a 4 percent drawdown, they both offer average utility scores of about 8.0. During worst-case scenarios, however, the *constant dollar* strategy can spell disaster, even at a level as low as 4 percent.

How Does the 4 Percent Rule Stack Up?

Our analysis shows that the 4 Percent Rule tends to be a reasonable baseline for consideration. Although the average utility score for a constant dollar strategy is highest at a 4.5 percent withdrawal rate, the best strategy for poor outcomes was found at a lesser rate of 3.5 percent.

As discussed above, a *constant dollar* strategy can quickly turn bad. Poor outcomes for *constant dollar* strategies that draw 5 percent or greater are significantly worse than the poor outcomes for *constant* or *increasing percentage* strategies. The worst-case utility scores for the *constant dollar* approach decrease significantly above a 3.5 percent drawdown. The *constant dollar* strategy is much more likely to cause depletion of assets than other strategies due to its inflexibility.

Constant dollar strategies that draw more than 4 percent can be unsafe in simulations with worse-than-expected return environments and for longer than average retirements.

Constant percentage strategies may allow for higher initial drawdown percentages, with the understanding that years of low income are possible. Generally, an *increasing percentage* strategy produces better outcomes than a *constant percentage* strategy. A conservative *constant dollar* strategy also outperforms a *constant percentage* strategy on average.

Impact of Investment Environment on Retirement Outcomes

The best drawdown percentage and strategy depends on the investment environment during retirement. During retirements starting between 1928 and 1974, the drawdown percentage that has maximized utility scores has fluctuated between 3 and 7.5 percent.

To exemplify the differences in retirement outcomes over time, first consider a drawdown starting in March 1965, as shown in Figure 3. (For ease of comparison, all examples assume a starting nest egg of $1 million.) This simulated retirement included weak asset returns and high inflation during the late 1960s and 1970s, resulting in outcomes among the poorest simulated. We applied three different strategies to this retirement period and compared the resulting annual income, as illustrated in the figure on page 18.

The first strategy starts with a 3 percent drawdown and adjusts the percentage upward with inflation each year. This is a relatively conservative approach. Over a 30-year period, the average annual drawdown is $34,395 and the minimum draw is $27,244 (5.98 utility score).

The second strategy uses a *constant dollar* approach based on a 4 percent drawdown. During this historical simulation, this strategy would have run out of assets after year 24 (4.67 utility score).

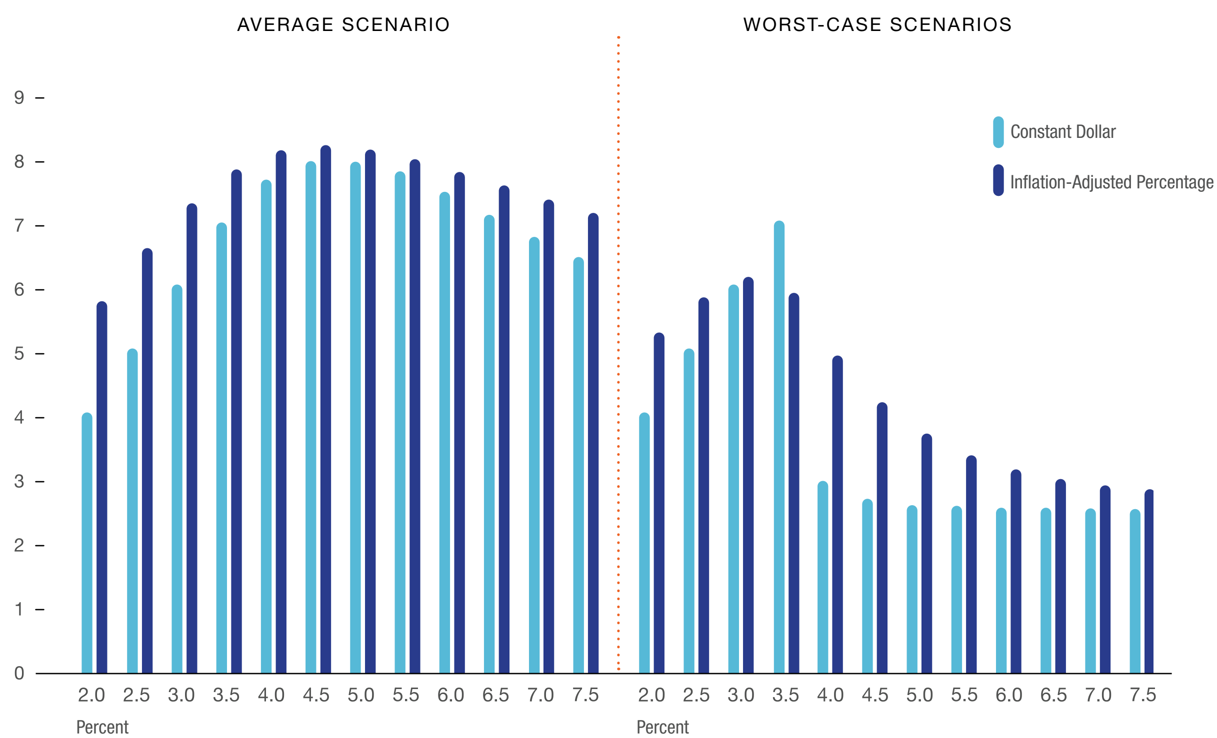
The third strategy in this example is a *constant percentage* approach with a 5 percent initial drawdown. This strategy resulted in higher drawdowns for the first nine years, but then dropped below the more conservative strategy due to lackluster asset returns. The average spending for the first 30 years in this simulation is $28,619 with a minimum annual spending of $14,524 in year 18 (4.48 utility score). This strategy allows for greater spending again in late retirement as asset returns turned upward, concurrent with the 1990s.

In this example, an *inflation-adjusted percentage* strategy with a 3 percent starting drawdown is preferable to a 4 percent *constant dollar* strategy and a 5 percent *constant percentage* strategy.

As a second example, consider the cohort retiring in July 1933, as shown in Figure 4. This post-depression, pre-WWII period saw relatively strong asset returns and modest inflation. For retirees in this cohort, a 3 percent *inflation-adjusted percentage* strategy was too conservative. The retiree could have spent a higher amount earlier in retirement and still not exhausted savings after 40 years. This simulation had 30-year annual spending of $46,064 and a minimum of $29,512 in the second year (7.33 utility score).

We applied a more aggressive constant 6 percent strategy to this retirement period

FIGURE 2. Average and Worst-Case Utility Score for Two Strategies



Source: Author's calculations.

FIGURE 3. Retirement in March 1965

ANNUAL INFLATION-ADJUSTED INCOME
FOR 3 EXAMPLE STRATEGIES

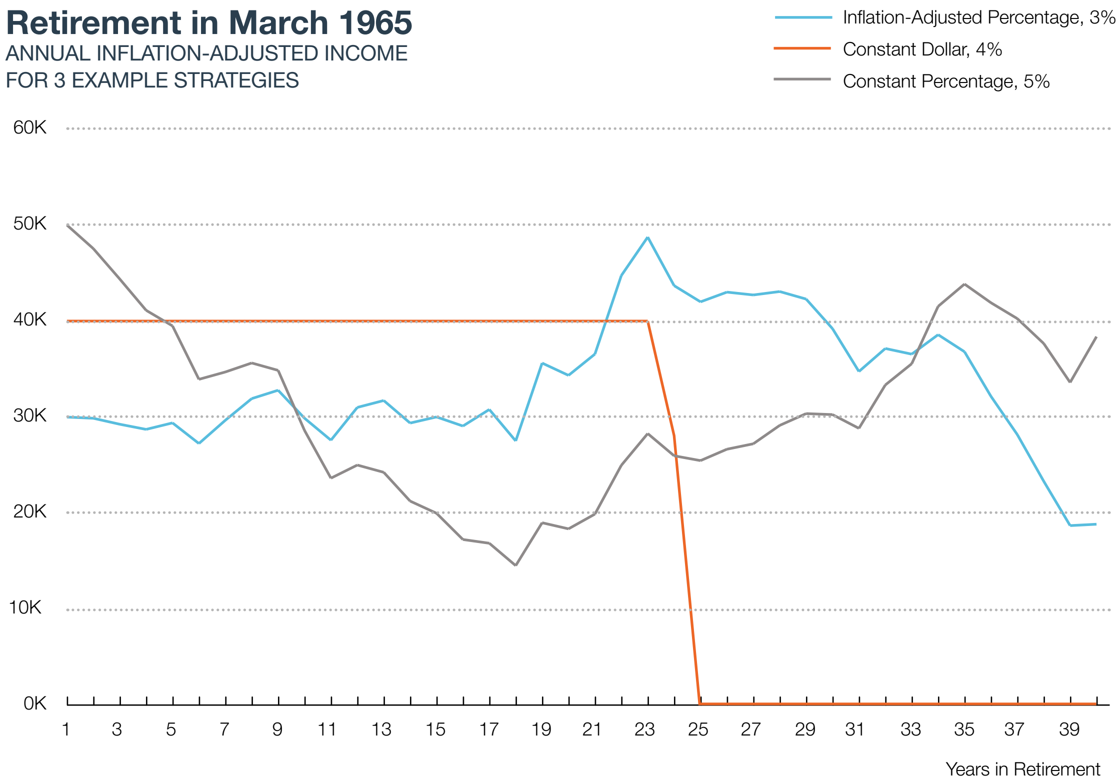
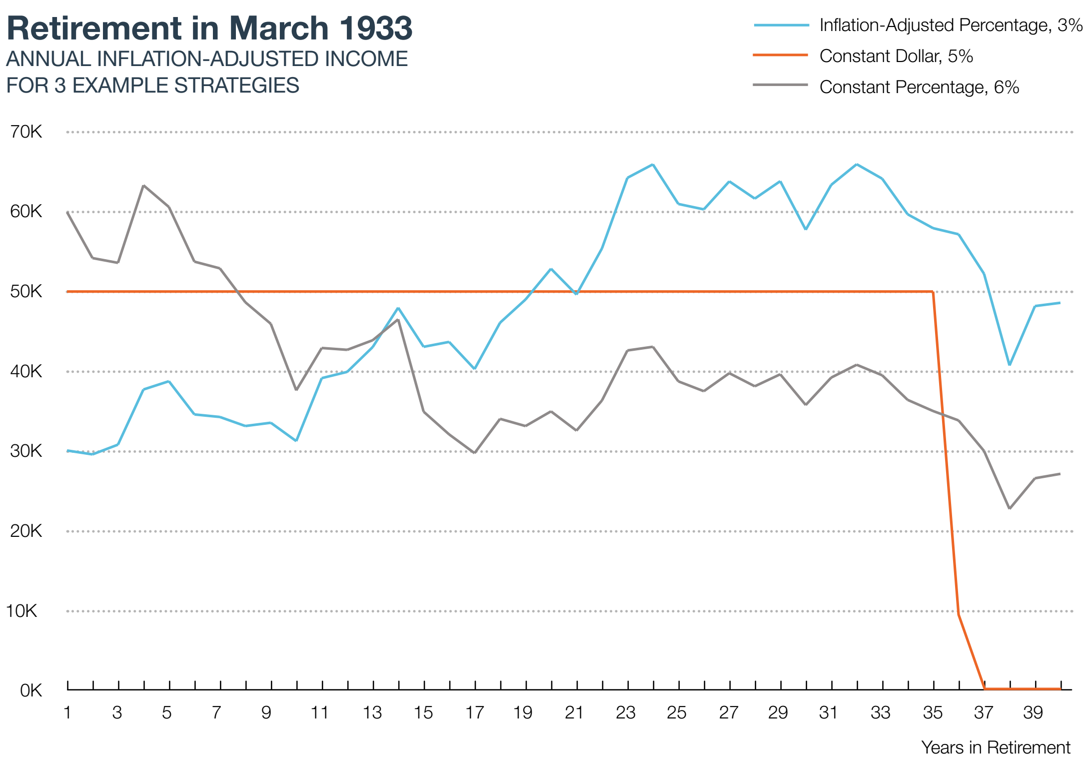


FIGURE 4. Retirement in March 1933

ANNUAL INFLATION-ADJUSTED INCOME
FOR 3 EXAMPLE STRATEGIES



and found that this would have been too aggressive during early retirement, resulting in lower drawdowns later in retirement. The average 30-year annual drawdown for this simulation was $42,963 with a minimum of $29,668 in year 17 (7.92 utility score).

Finally, a strategy that drew a constant, inflation-adjusted $50,000 would have allowed the portfolio to survive 35 years. This represents the best possible simulation tested for this cohort (9.44 utility score). Despite its lack of flexibility and relative aggressiveness, the retirement period returns were such that this strategy maximized utility. However, a retirement lasting more than 35 years would have run out of assets using this strategy.

These are just two examples that demonstrate how the investment period plays a critical role in retirement outcomes. Appendix C offers average outcomes over all the historical simulations.

Making Midterm Adjustments

Simulations represent fixed strategies. The reality is that plans can be adjusted based on market returns and life changes. A 3.5 percent drawdown during the first five years of retirement may result in a growing portfolio. If this happens, options abound for the remaining retirement. On the other hand, a 5 percent drawdown during the first several years may be met with poor market returns and a worse-than-expected outcome. Under these circumstances, the strategy may need to be recalibrated. Perhaps a 3 percent drawdown for the following five years would be an appropriate adjustment.

Retirement need not be considered as a set period of time. It can be grouped into decades, five-year periods, or even single years. If the first one, five, or ten years of retirement offer unforeseen returns, adjustments are possible. It is unreasonable to believe that the initial strategy will still fit circumstances in 20 years. Finances should be reviewed annually.

Other Retirement Drawdown Factors

This report has offered a starting point, looking at two influential factors: drawdown strategy and drawdown percentage. Is there an optimal and implementable strategy for drawing down assets in retirement? Is there an optimal level of spending? There are additional factors that influence retirement outcomes that are not covered by this model but should be mentioned here. Future efforts will be made to extend the analysis to these factors:

Pre-Retirement Returns: Randomness and timing strongly influence outcomes in historical simulations. The model does not, however, adjust the starting portfolio based on pre-retirement returns. When returns prior to retirement are particularly high, mean reversion suggests that retirements following strong returns should anticipate weaker returns in the near future, suggesting more modest drawdowns. Recent returns and asset accumulation are important considerations for determining optimal withdrawal rates. Because of this, Pfau posits that a higher-than-expected accumulation during pre-retirement may mandate a lower drawdown percentage, whereas a lower-than-expected accumulation of assets could mean that a higher drawdown percentage would be sustainable.[xix] He links pre-retirement accumulation to retirement outcomes.

Asset Allocation: There is considerable research on the optimal asset allocation strategy in retirement. Bengen finds that a 50-75 percent equity allocation allows for retirement success during a majority of historical simulations. A subsequent study finds that “most retirees would likely benefit from allocating at least 50 percent to common stocks.”[xx]

Other recent efforts have suggested a variety of asset allocations. The literature has generally moved toward more aggressive equity allocations (50+ percent). This is the result of modeling techniques. Retirement drawdown models usually rely on historical returns. The pattern of historical stock and bond returns is such that over sufficiently long periods of time, a higher constant allocation to equities is almost always preferable. If the goal is to reduce the probability that retirement assets are exhausted, a more aggressive portfolio has historically outperformed on this metric.

Another quantitative study finds a 30-year failure rate of 67.4 percent for conservative portfolios (20 percent stock) as compared with an 8.4 percent 30-year failure rate for aggressive portfolios (85 percent stock).[xxi] The tradeoff is a higher likelihood of failure early in retirement. For a 20-year retirement, the conservative portfolio fails 0.9 percent of the time, whereas an aggressive allocation fails 1.7 percent of the time.

Pfau proposes a more dynamic approach that challenges conventional wisdom. His strategy suggests a modest allocation to equities early in retirement with a more aggressive approach as the retiree ages.[xxii] This research supports maintaining a modest drawdown early in retirement.

Return Assumptions: Historical data are used for return assumptions, understanding that a reduced equity premium in the future would result in less desirable outcomes. Future efforts will be made to model alternative return assumptions.

Annuitization and Pension Assets: Research suggests that a strategy that annuitizes all or a portion of retirement assets may improve outcomes. One such study posits that the optimal decision includes a higher allocation to equities and 25-50 percent annuitization.[xxiii] Several papers find that greater pension assets (including Social Security) may allow a higher spending percentage of financial assets.[xxiv,xxv] Pension assets are not included in this analysis.

Bequest Motive: The analysis assumes that there is no bequest motive, meaning that the remaining amount at the end of retirement is inconsequential to the utility score.

Taxes and Fees: Taxes and fees are excluded from the analysis. That does not mean they should be ignored. Fees are one of the most controllable parameters in retirement financing. A simple index fund can incur fees of less than 0.2 percent. More complex mutual funds and other products can range in fees from 0.5 percent to more than 2 percent. Fees are a critical consideration in retirement finance.

05 Conclusion

Retirees face four major retirement finance risks in determining a drawdown strategy: the uncertainty of lifespans, fluctuating market returns, potential inflation, and unforeseen personal expenditures. The factors influencing these risks and their effect on retirement outcomes are difficult for retirees to control or predict. For example, a retiree from 1979 had the good fortune of excellent market returns during the 1980s and 1990s. A retiree from 1964, on the other hand, dealt with low returns and inflation through much of early retirement.

Selecting a long-term strategy for withdrawing retirement assets can be a daunting task. Many studies aim to identify an optimal strategy, only to offer a variety of conclusions. Historical simulations can help instruct a secure retirement withdrawal strategy.

The first five to 10 years of retirement are critical to long-term outcomes. When the first 10 years of retirement go well and the nest egg remains robust, options abound for the remainder of retirement. One general rule is to spend as little as is comfortable early in retirement. During later retirement years, spending flexibility can help improve outcomes. Keep in mind that returns are rarely average.

Some near-retirees are looking for a strategy that will allow 6, 8, or 10 percent annual drawdowns. This may be possible with some terrific luck, but historical returns do not support these drawdown amounts. Retirees that seek to draw high percentages early in retirement likely need to think about how to reduce spending later in retirement or to supplement income through other sources.

This study finds that 3-5 percent initial drawdown has been historically ideal. Outcomes are also maximized when flexible strategies (a changing percentage of assets) are employed, as compared with the traditional *constant dollar* strategy. A *constant dollar* strategy can be useful if it provides comfort. It may be prudent to revisit the strategy every few years, however, to ensure savings are not dwindling beyond a reasonable threshold.

Retirement finance is as much art as science, an art that can be practiced by retirees themselves. A retirement strategy that is informed by quantitative analysis and is adaptive to market and life conditions is most likely to provide financial security.

Appendix A: Estimating Utility

Utility is the satisfaction a person harvests from consumption. A utility function is a mathematical formula created to quantify the qualitative experience of that satisfaction, assigning a numerical value to utility. The utility function includes only two components, which are added together to determine that numerical value:

> **Average Spending Percentage:** Utility is positively impacted when a strategy allows the retiree to spend a higher amount throughout retirement. The average spending rate is adjusted for inflation and calculated as a percentage of the original nest egg. If the spending rate starts high, utility may be severely penalized when the nest egg runs out. The average is calculated based on weighted average life expectancy for a couple. If the nest egg runs out early, the utility score will be significantly reduced because the years of zero annual income will reduce the average. The average spending component is maximized when it appropriately balances the risk of running out of retirement assets with the reward of high spending.
>
> **Minimum Spending:** The second component of the utility metric is the lowest yearly level of spending as a percentage of the original nest egg. In the case where annual income approaches zero, this component affects utility in a negative way. This component is meant to balance the average spending percentage. When a moderate, constant starting spending percentage, such as 4 percent, is maintained throughout retirement, utility would equal 8.00 (4 percent average + 4 percent minimum).

Each of the strategies is simulated with a range of starting drawdown amounts from 2 percent to 7.5 percent. The utility function is constructed simply as follows:

Utility = ((Average Spending %) + (Minimum Spending %))

As a measurement example, suppose a strategy draws 4.5 percent of assets using a *smoothed percentage* strategy. The first simulation uses monthly stock, bond, and inflation returns from January 1928 through December 1967. Assume a starting portfolio value of $1 million. In the first year, the simulation draws $45,000. Markets rise and the value of the portfolio increases to about $1.15 million. The second year sees a drawdown of about $48,375 (the average of $45,000 and $51,750). Over a 30-year simulation, the average annual drawdown is an inflation-adjusted $40,800. The minimum drawdown is $30,300, in the 22nd year. The 30-year utility score of this outcome would be:

Average Spending (4.08) + Minimum Spending (3.03) = 7.11

For comparative purposes, this would be equivalent to drawing down 3.555%, inflation adjusted, in every year:

Average Spending (3.555) + Minimum Spending (3.555) = 7.11

These two strategies leave different amounts of bequest, but it is simple to see the relative relationship between outcomes. Where the first strategy has a higher average spending amount, it also has a higher level of variability.

Although the utility function is a construct, it is meant to portray the general tradeoff between spending and variability. It should be noted that variability to the upside (increasing amounts) does not reduce the utility score. However, the average spending rate and minimum spending amounts could be diminished by using a starting drawdown that is too low.

There are six measures of utility provided in Appendix C:

Average Retirement: This metric averages utility across all 553 simulations. Each simulation uses a weighted average retirement length based on mortality assumptions (See Appendix B).

Long Retirement: This metric assumes a 35-year retirement and averages utility across simulations. It is meant to represent a longer than normal retirement period.

Poor Outcomes: This metric takes the 5th percentile of weighted average utilities across simulations. It is meant to represent poor historical outcomes.

Worst-Case Scenario: This metric examines the 5th percentile of simulations for 35-year retirements. This represents long retirements with poor outcomes, a worst-case scenario.

Average Utility – Low 7-Year Stock Returns: This metric averages utility over the 56 simulations with the poorest initial seven-year stock returns, which is useful for considering below-average stock return environments.

Average Utility – Low 5-Year Bond Returns: This metric averages utility over the 56 simulations with the poorest initial five-year bond returns, which is useful for considering below-average bond return environments.

Appendix B: Simulation Methodology

The simulations include 553 historical 40-year periods based on monthly return data from January 1928 through December 2013. There are limitations to historical analysis. First, the simulations are not independent. For instance, returns for December 1928 will appear in all of the first 12 simulations. The months in the middle of the series will be overrepresented as compared with the months at the beginning and end of the series. For instance, January 1928 and December 2013 will exist in only the first and last simulation, whereas a mid-series month will exist in up to 480 simulations. Nonetheless, this analysis offers a unique comparison of historical retirement timing.

A 50 percent stock and 50 percent bond allocation for every simulation is assumed for simplicity. The purpose of this analysis is not to offer asset allocation guidelines but to isolate the effect of drawdown amounts and strategies.

Each strategy and drawdown percentage is simulated across each of the 553 return streams. The strategy, initial drawdown percentage, and simulated returns generate annual drawdown amounts and the remaining savings during any month. Annual withdrawals are used to calculate utility.

Data: Monthly stock returns data are from the S&P 500 total return index, downloaded from Bloomberg. Monthly bond data are Long Term Corporate Bond returns from Morningstar and Ibbotson. Inflation data are based on the Consumer Price Index from the Bureau of Labor Statistics.

Historical Simulated Returns: 553 individual return streams are created from chronological 40-year periods. The first simulation uses returns from January 1928 through December 1967. The second simulation uses returns from February 1928 through January 1968. The 553rd simulation uses data from January 1974 through December 2013. Return streams are highly correlated. For instance, the first and second simulation will have nearly identical results. Nonetheless, the simulated returns offer a wide variety of outcomes. The randomness generated through historical simulation provides results that are similar to the Monte Carlo simulation, which randomizes return streams (see Appendix D).

Simulation Process: A starting nest egg is chosen for simulation ($1 million). The starting amount is inconsequential because drawdowns are based on some percentage of the starting amount. The nest egg is recalculated on a monthly basis accounting for market returns and drawdown amounts.

Drawdowns are calculated on an annual basis and taken prior to the first month of the year. The first drawdown occurs prior to any monthly returns. For example, if a $40,000 drawdown is calculated from a $1,000,000 portfolio, the first month's return will be on only $960,000.

The asset allocation (50 percent stocks, 50 percent bonds) is rebalanced on an annual basis. During months 1-12, asset allocation will float based on market returns. Prior to month 13, and every subsequent 12 months, the portfolio is rebalanced to 50/50.

Drawdown amounts are based on the strategy chosen for simulation, the value of the nest egg at the end of each year, and inflation. For example, if a 5 percent *inflation-adjusted percentage* strategy is employed with a starting value of $1,000,000, the first drawdown will be $50,000. If the value of the total portfolio is $1,100,000 after 12 months and inflation was 2 percent for the year, the drawdown prior to month 13 will be calculated as: 5 percent x 1.02 x 1,100,000 = $56,100. The remaining $1,043,900 would then be subject to the following 12 months of market returns (starting with $521,950 each in stocks and bonds).

Utility Calculation: For each simulation, utility scores are calculated for each length of retirement from one to 40 years. Utility is the average spending amount plus the minimum spending amount divided by the starting amount. For example, for a 20-year retirement with average spending of $47,000 and minimum spending of $36,000 on a $1,000,000 original portfolio, the utility measure would be 4.7 + 3.6 = 8.3.

Average utility scores for a given simulation are provided by taking a weighted average of utility scores based on life expectancy. Joint mortality expectations are used, which assumes a married couple and assumes that utility scores will be calculated in the same manner when only one spouse is present. Weights are based on Social Security Administration mortality assumptions and assume retirement beginning at age 65. The weights used are in the table below:

Retirement Length	% Likelihood	Retirement Length	% Likelihood	Retirement Length	% Likelihood
<=11	2.73	21	3.23	31	5.15
12	0.78	22	3.64	32	5.17
13	0.94	23	4.16	33	4.70
14	1.12	24	4.47	34	4.13
15	1.34	25	4.87	35	3.50
16	1.58	26	5.23	36	2.86
17	1.85	27	5.52	37	2.26
18	2.14	28	5.71	38	1.73
19	2.47	29	5.78	39	1.29
20	2.83	30	5.72	40+	2.85

We also present utility scores for a 35-year retirement length. This assumes a 100 percent weight on the 35-year outcome. This is meant to represent a longer than average retirement period, because only about 14.5 percent of retirements last at least 35 years (past age 65).

Required Minimum Distribution Percentage (RMD) Assumptions: The following table provides the percentage drawdown for each year after age 70 under the RMD simulations:

Age	% Drawdown	Age	% Drawdown	Age	% Drawdown
70	3.65	80	5.35	90	8.77
71	3.77	81	5.59	91	9.26
72	3.91	82	5.85	92	9.80
73	4.05	83	6.13	93	10.42
74	4.20	84	6.45	94	10.99
75	4.37	85	6.76	95	11.63
76	4.55	86	7.09	96	12.35
77	4.72	87	7.46	97	13.16
78	4.93	88	7.87	98	14.08
79	5.13	89	8.33	99	14.93

Appendix C: Historical Utility Scores

Average Utility Scores

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	4.21	4.05	3.49	4.70	5.75	6.34	6.89
2.5%	5.00	4.95	4.79	4.27	5.66	6.58	7.11	7.31
3.0%	6.00	5.57	5.42	4.98	6.57	7.28	7.62	7.61
3.5%	6.97	6.08	5.96	5.63	7.44	7.81	7.88	7.76
4.0%	7.64	6.49	6.40	6.20	8.00	8.11	7.99	7.80
4.5%	7.93	6.81	6.76	6.66	8.17	8.19	7.97	7.77
5.0%	7.92	7.05	7.04	7.01	8.07	8.12	7.87	7.69
5.5%	7.77	7.22	7.25	7.24	7.78	7.97	7.71	7.59
6.0%	7.45	7.33	7.39	7.39	7.39	7.77	7.56	7.49
6.5%	7.09	7.40	7.47	7.46	7.00	7.56	7.42	7.39
7.0%	6.75	7.42	7.48	7.48	6.63	7.34	7.29	7.29
7.5%	6.43	7.39	7.44	7.45	6.32	7.13	7.17	7.19

35-Year Utility Scores— Long Retirement

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	4.47	4.28	3.49	4.94	6.47	7.14	7.47
2.5%	5.00	5.15	4.98	4.26	5.83	7.11	7.55	7.84
3.0%	6.00	5.67	5.53	4.97	6.66	7.61	7.64	8.08
3.5%	6.90	6.06	5.95	5.60	7.39	7.87	7.34	8.16
4.0%	7.21	6.32	6.25	6.11	7.53	7.59	6.99	8.10
4.5%	7.02	6.47	6.45	6.46	7.13	7.03	6.64	7.98
5.0%	6.45	6.54	6.54	6.61	6.53	6.48	6.31	7.84
5.5%	5.97	6.54	6.57	6.63	5.59	6.02	6.04	7.70
6.0%	5.03	6.48	6.53	6.58	4.78	5.63	5.83	7.55
6.5%	4.56	6.39	6.43	6.48	4.18	5.29	5.66	7.41
7.0%	4.14	6.26	6.28	6.34	4.00	5.00	5.53	7.26
7.5%	3.96	6.11	6.11	6.18	3.89	4.75	5.43	7.12

5th Percentile Utility Scores — Poor Outcomes

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	2.96	2.78	2.72	4.10	4.40	4.97	5.35
2.5%	5.00	3.43	3.25	3.25	5.04	5.14	5.44	5.57
3.0%	6.00	3.81	3.66	3.71	6.00	5.72	5.72	5.53
3.5%	6.92	4.13	3.99	4.07	6.90	6.02	5.84	5.49
4.0%	5.23	4.38	4.27	4.36	5.22	5.99	5.89	5.44
4.5%	4.35	4.59	4.50	4.59	4.35	5.80	5.84	5.39
5.0%	4.04	4.76	4.68	4.76	4.04	5.48	5.70	5.35
5.5%	3.95	4.89	4.83	4.89	3.95	5.18	5.54	5.31
6.0%	3.84	4.98	4.94	4.98	3.84	4.94	5.40	5.25
6.5%	3.81	5.06	5.02	5.06	3.81	4.74	5.31	5.19
7.0%	3.77	5.11	5.08	5.11	3.77	4.59	5.24	5.13
7.5%	3.74	5.14	5.12	5.14	3.74	4.46	5.20	5.07

35-Year 5th Percentile Utility Scores — Worst-Case Scenario

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	3.27	3.03	2.78	4.25	5.26	5.73	5.90
2.5%	5.00	3.71	3.48	3.33	5.09	5.81	6.03	6.19
3.0%	6.00	4.05	3.84	3.79	6.00	6.13	5.97	6.10
3.5%	7.00	4.31	4.12	4.18	7.00	5.88	5.74	6.01
4.0%	2.93	4.49	4.33	4.45	2.93	4.90	5.53	5.92
4.5%	2.65	4.62	4.48	4.62	2.65	4.17	5.25	5.82
5.0%	2.55	4.70	4.58	4.71	2.55	3.68	4.99	5.71
5.5%	2.54	4.74	4.64	4.75	2.54	3.34	4.80	5.61
6.0%	2.51	4.76	4.66	4.76	2.51	3.12	4.65	5.52
6.5%	2.51	4.74	4.65	4.75	2.51	2.97	4.54	5.44
7.0%	2.50	4.72	4.62	4.72	2.50	2.87	4.45	5.35
7.5%	2.49	4.67	4.57	4.67	2.49	2.81	4.38	5.25

Average Utility Scores — Low 7-Year Stock Returns

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	4.00	4.15	3.39	4.57	4.45	5.96	6.67
2.5%	5.00	4.72	4.92	4.18	5.49	5.15	6.63	7.12
3.0%	6.00	5.37	5.56	4.94	6.36	5.75	7.16	7.49
3.5%	7.00	5.93	6.06	5.63	7.23	6.25	7.56	7.70
4.0%	7.79	6.39	6.45	6.23	7.91	6.66	7.82	7.72
4.5%	8.32	6.75	6.74	6.69	8.39	7.00	7.85	7.61
5.0%	8.35	7.01	6.95	7.01	8.37	7.27	7.70	7.48
5.5%	7.68	7.19	7.10	7.20	7.63	7.49	7.50	7.36
6.0%	6.80	7.29	7.18	7.31	6.73	7.66	7.32	7.23
6.5%	6.20	7.34	7.20	7.35	6.16	7.76	7.17	7.11
7.0%	5.84	7.34	7.13	7.35	5.82	7.77	7.03	6.98
7.5%	5.60	7.29	7.03	7.31	5.59	7.70	6.92	6.86

Average Utility Scores — Low 5-Year Bond Returns

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	3.70	3.53	3.29	4.40	5.36	5.65	6.10
2.5%	5.00	4.30	4.14	3.97	5.37	6.13	6.35	6.46
3.0%	6.00	4.79	4.65	4.55	6.33	6.77	6.78	6.68
3.5%	6.77	5.18	5.08	5.03	7.06	7.22	6.92	6.81
4.0%	6.58	5.49	5.42	5.42	6.82	7.34	6.95	6.87
4.5%	6.56	5.74	5.70	5.72	6.73	7.25	6.86	6.86
5.0%	6.62	5.92	5.91	5.94	6.62	7.08	6.74	6.80
5.5%	6.59	6.06	6.07	6.10	6.46	6.89	6.62	6.73
6.0%	6.45	6.16	6.19	6.20	6.36	6.71	6.52	6.66
6.5%	6.32	6.23	6.26	6.27	6.25	6.54	6.43	6.59
7.0%	6.17	6.26	6.30	6.30	6.10	6.39	6.34	6.53
7.5%	6.02	6.27	6.31	6.30	5.96	6.25	6.28	6.46

Appendix D: Monte Carlo Simulations

The findings are corroborated by results from a Monte Carlo simulation. In the Monte Carlo simulation, months are selected at random from January 1928 through December 2013 with replacement to create 1,000 unique return streams (a bootstrapping methodology). The Monte Carlo simulation does not account for return momentum. The simulations assume that monthly returns are independently distributed. Although the historical method is preferred for its relative simplicity, the Monte Carlo simulation is helpful in corroborating the findings.

The Monte Carlo simulations tend to result in higher average utility scores. This is because strategies tend to fair best when returns are random and uncorrelated. Due to the randomness of return selection, a Monte Carlo simulation is unlikely to replicate, for instance, a stretch of high inflation such as that experienced during the 1970s. Likewise, it is unlikely that the simulations would replicate many negative returns for consecutive months, although these trends are prominent in the historical data. Nonetheless, the simulations offer a similar pattern to the historical simulations. The tables on the following pages offer the utility scores measured through the Monte Carlo analysis and illustrate the following points:

- Monte Carlo simulation finds a 5 percent *inflation-adjusted percentage* strategy to be optimal on average.
- The required minimum distribution strategy is optimal during longer retirements.
- For poor outcomes, a 3.5 percent *inflation-adjusted percentage* strategy is optimal.
- Low seven-year stock returns have a severe effect on outcomes in the Monte Carlo simulation.
- Monte Carlo simulation results in more favorable outcomes than historical simulation on average.

Average Utility Scores

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	4.96	4.76	3.70	5.25	6.48	7.43	8.02
2.5%	4.99	5.84	5.65	4.55	6.27	7.42	8.22	8.45
3.0%	5.96	6.59	6.44	5.35	7.20	8.18	8.78	8.81
3.5%	6.88	7.24	7.12	6.10	8.02	8.79	9.13	9.06
4.0%	7.68	7.77	7.71	6.77	8.69	9.24	9.29	9.17
4.5%	8.31	8.19	8.18	7.37	9.17	9.52	9.31	9.17
5.0%	8.76	8.51	8.55	7.86	9.45	9.63	9.23	9.11
5.5%	8.98	8.73	8.82	8.25	9.50	9.61	9.09	9.00
6.0%	8.95	8.86	8.99	8.54	9.31	9.47	8.92	8.88
6.5%	8.77	8.92	9.07	8.73	8.98	9.25	8.75	8.75
7.0%	8.45	8.92	9.08	8.82	8.55	8.98	8.59	8.61
7.5%	8.06	8.86	9.01	8.84	8.07	8.68	8.45	8.47

35-Year Utility Scores — Long Retirement

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	5.45	5.22	3.69	5.74	7.52	8.68	9.00
2.5%	4.97	6.29	6.11	4.53	6.71	8.30	8.98	9.36
3.0%	5.91	6.97	6.84	5.32	7.55	8.84	9.00	9.65
3.5%	6.76	7.51	7.44	6.04	8.22	9.18	8.81	9.83
4.0%	7.33	7.91	7.90	6.67	8.57	9.30	8.49	9.88
4.5%	7.68	8.18	8.23	7.19	8.71	9.16	8.13	9.82
5.0%	7.81	8.32	8.42	7.58	8.57	8.83	7.78	9.70
5.5%	7.54	8.34	8.49	7.85	8.06	8.35	7.45	9.54
6.0%	7.20	8.28	8.46	8.00	7.45	7.79	7.18	9.36
6.5%	6.73	8.15	8.33	8.02	6.83	7.21	6.96	9.17
7.0%	6.18	7.95	8.12	7.94	6.09	6.66	6.77	8.98
7.5%	5.69	7.72	7.84	7.78	5.54	6.19	6.62	8.79

5th Percentile Utility Scores — Poor Outcomes

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	3.01	3.01	2.89	4.02	4.01	4.60	5.14
2.5%	5.00	3.48	3.49	3.42	5.00	4.63	5.20	5.44
3.0%	6.00	3.91	3.89	3.87	6.00	5.11	5.53	5.40
3.5%	6.30	4.26	4.25	4.24	6.30	5.43	5.68	5.34
4.0%	5.20	4.56	4.55	4.52	5.20	5.53	5.57	5.29
4.5%	4.71	4.74	4.75	4.74	4.71	5.57	5.48	5.24
5.0%	4.56	4.89	4.87	4.89	4.56	5.57	5.34	5.19
5.5%	4.33	5.01	4.95	5.01	4.33	5.54	5.21	5.15
6.0%	4.18	5.08	5.03	5.08	4.18	5.35	5.15	5.11
6.5%	4.05	5.11	5.06	5.11	4.05	5.25	5.07	5.05
7.0%	3.99	5.09	5.05	5.09	3.99	5.16	4.97	4.99
7.5%	3.91	5.06	5.00	5.06	3.91	5.04	4.90	4.93

35-Year 5th Percentile Utility Scores — Worst-Case Scenario

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	2.99	2.99	2.86	4.03	4.28	4.96	5.23
2.5%	5.00	3.41	3.41	3.34	5.00	4.76	5.10	5.37
3.0%	6.00	3.74	3.74	3.71	6.00	4.98	4.80	5.32
3.5%	3.31	3.94	3.95	3.93	3.31	5.02	4.46	5.26
4.0%	2.92	4.11	4.10	4.11	2.92	4.93	4.27	5.20
4.5%	2.81	4.20	4.18	4.20	2.81	4.65	4.11	5.10
5.0%	2.81	4.21	4.20	4.21	2.81	4.39	3.98	5.04
5.5%	2.75	4.21	4.17	4.21	2.75	4.15	3.89	4.98
6.0%	2.70	4.18	4.09	4.18	2.70	3.94	3.81	4.91
6.5%	2.65	4.13	4.01	4.13	2.65	3.73	3.74	4.83
7.0%	2.64	4.04	3.91	4.05	2.64	3.56	3.70	4.75
7.5%	2.60	3.98	3.83	3.99	2.60	3.42	3.66	4.69

Average Utility Scores — Low 7-Year Stock Returns

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	2.30	1.98	1.94	1.77	2.49	2.64	3.06	3.34
2.5%	2.82	2.32	2.29	2.12	2.98	3.02	3.40	3.54
3.0%	3.28	2.60	2.59	2.44	3.40	3.32	3.63	3.59
3.5%	3.58	2.84	2.84	2.71	3.66	3.55	3.74	3.57
4.0%	3.74	3.03	3.04	2.94	3.77	3.70	3.76	3.53
4.5%	3.69	3.18	3.20	3.12	3.70	3.77	3.71	3.49
5.0%	3.45	3.29	3.32	3.25	3.45	3.78	3.63	3.45
5.5%	3.17	3.37	3.39	3.34	3.16	3.74	3.54	3.41
6.0%	2.89	3.42	3.43	3.40	2.89	3.68	3.47	3.37
6.5%	2.68	3.44	3.44	3.43	2.68	3.59	3.40	3.32
7.0%	2.55	3.44	3.43	3.43	2.55	3.49	3.34	3.28
7.5%	2.46	3.42	3.39	3.42	2.46	3.38	3.29	3.24

Average Utility Scores — Low 5-Year Bond Returns

	Constant Dollar	Constant Percentage	Smoothed Percentage	Constant Percentage Ceiling	Constant Percentage Floor	Inflation-Adjusted Percentage	Increasing Percentage	Required Minimum Distribution
2.0%	4.00	4.19	4.06	3.43	4.72	5.51	6.30	6.88
2.5%	4.99	4.92	4.82	4.19	5.68	6.32	7.00	7.27
3.0%	5.94	5.55	5.47	4.89	6.55	6.98	7.50	7.52
3.5%	6.72	6.09	6.04	5.52	7.23	7.49	7.80	7.61
4.0%	7.24	6.52	6.51	6.07	7.64	7.86	7.91	7.58
4.5%	7.61	6.85	6.87	6.51	7.91	8.06	7.87	7.50
5.0%	7.70	7.10	7.14	6.85	7.89	8.13	7.75	7.40
5.5%	7.47	7.26	7.32	7.11	7.60	8.06	7.58	7.30
6.0%	7.01	7.36	7.43	7.27	7.08	7.89	7.41	7.19
6.5%	6.54	7.39	7.46	7.35	6.58	7.67	7.25	7.08
7.0%	6.12	7.38	7.44	7.37	6.12	7.42	7.10	6.97
7.5%	5.78	7.33	7.36	7.34	5.77	7.16	6.99	6.86

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