Co-Movements between Financial Markets and the Real Economy

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Abstract

JEL codes: C32, E32, G17
Key words: Dynamic factor models, financial markets, co-movements

The long-term relationship between financial markets and economic conditions is unclear. This paper aims to investigate how movements in the financial markets interact with the broader economy in the long term. Dynamic factor models are implemented to capture unobserved factors—common factor and sector factors. The common factor represents the co-movement between the real economy and the financial markets, and the sector factors indicate co-movements within the economy or within the financial markets. Variance decomposition is performed to show how much of variation in each variable can be explained by the co-movements. The results show that bond indexes are highly co-moved with money/credit related economic indicators, but stock indexes seem only to co-move with one another, and a big portion of variation in the stock market remains unexplained.

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1. Introduction

Financial markets are well known to be volatile in the short term, making day-to-day market movements unpredictable. In the medium term, however, especially during transitional periods in the business cycle, financial markets appear to move in relation with economic conditions. But in the longer term, such a relationship becomes unclear again. This paper is aimed at capturing such co-movements between the real economy and financial markets in the past three decades. If such co-movements are found to be significant, they can provide long-term investors with useful guidance on how to select their investment portfolios based on related economic variables. Moreover, the investigation of co-movements among financial instruments helps investors to hedge risk by avoiding co-moved instruments in their portfolios.

Correlations between economic conditions and bond and stock returns have attracted enormous attention in the literature. Most researchers have found important correlations between stocks and bonds, or cycles within financial markets, but there is a lack of evidence of co-movements between economic growth and financial market performance. Fama and French (1989) attempt to answer whether the expected returns on bonds and stocks move together and whether the variation in expected bond and stock returns is related to business conditions. They run F-tests to conclude that movement in expected returns is largely common across securities and is negatively related to long- and short-term business conditions; i.e., expected returns are lower when economic conditions are strong and higher when conditions are weak.

Cochrane (1991) investigates production-based asset pricing and the link between stock returns and economic fluctuations. The main finding is that ex post investment returns and stock returns are highly correlated and that the projection of investment and stock returns on investment/capital ratios matches in many respects.

Campbell and Ammer (1993) look at variability in long-term asset returns, interest rates, inflation, and other variables that may explain variability in stock and bond prices. Pindyck and Rotemberg (1993) first run OLS regressions of stock returns on current and lagged values of macroeconomic variables. The results show that the unexplained movements in returns remain excessively correlated. Connolly, Stivers, and Sun (2005) attempt to find the co-movement between the unexpected component of daily bond and stock returns by performing VAR models. The main results suggest that bond returns tend to be high (low) during periods when stock volatility increases (decreases).

All the above-mentioned research is performed under assumptions. For instance, VAR models assume that financial markets’ performance is based on current and lagged values. Assumptions, by their nature, restrict the realm of possible connection that can be investigated. Also, very few existing studies on this subject include the financial crisis of 2007 in their sample. In this paper, we relax all assumptions by implementing a dynamic factor model to capture unobserved co-movements between economic variables and financial benchmark securities over the period of 1987-2014.

We follow the model framework that has performed well in business cycle studies. Kose, Otrok, and Whiteman (2003) make an important contribution to the literature on international business cycles. The authors use Bayesian dynamic latent factor model to estimate common components in macroeconomic aggregates (output, consumption, and investment) of various countries, in order to study multiple co-movements simultaneously. Del Negro and Otrok (2008) extend the work by introducing time-varying parameters to the dynamic factor models.
The aim of this paper is to investigate whether co-movements exist between economic conditions and financial market performance in the U.S. during the period of 1987-2014. Variance decomposition is performed to capture the percentage of variation in each selected variable explained by co-movements.

The rest of the paper is organized as follows: Section 2 describes the data; Section 3 introduces the model setup and discusses empirical methodologies; Section 4 analyzes the empirical results; Section 5 concludes and discusses implications. Appendices and references are listed at the end.

2. Data

The dataset contains monthly data on twelve variables: six variables from the financial market and six variables from the real economy. Data on stock market returns are obtained through Federal Reserve Economic Data (FRED) and Yahoo finance. Bond market returns come from Barclays and the Dimensional Returns database. Data on macroeconomic variables come from Federal Reserve Economic Data (FRED). These variables were observed monthly from October 1987 to May 2014. The variable description is presented in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
<tr>
<td><strong>Financial Market: Stocks</strong></td>
<td></td>
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<tr>
<td>S&amp;P 500</td>
<td>Includes returns from 500 large companies’ common stocks (large capitalization)</td>
</tr>
<tr>
<td>NASDAQ-100</td>
<td>Includes returns from 100 of the largest non-financial companies listed on the NASDAQ</td>
</tr>
<tr>
<td>Russell 2000</td>
<td>Includes returns from 2000 small-cap to mid-cap companies</td>
</tr>
<tr>
<td><strong>Financial Market: Bonds</strong></td>
<td></td>
</tr>
<tr>
<td>Barclays U.S. Government/Credit Index</td>
<td>Includes a broad-based index composed of both government and corporate debt issues that are investment grade (rated Baa/BBB or higher).</td>
</tr>
<tr>
<td>Barclays U.S. Treasury Bond Index</td>
<td>The measure of the public obligations of the U.S. Treasury. Includes: public obligations of the U.S. Treasury, Fixed-rate bullet, puttable, and callable bonds, Soft bullets</td>
</tr>
<tr>
<td>Barclays U.S. High-Yield Bond Index</td>
<td>Represents USD-denominated, non-investment grade, fixed-rate, taxable corporate bond market. Securities are classified as high yield if the middle rating of Moodys, Fitch, and S&amp;P is Ba1/ BB+/BB+ or below</td>
</tr>
</tbody>
</table>
Macroeconomic variables

M2 Money Supply
Includes M1 money supply (notes and coins in circulation, traveler’s checks, demand deposits and other checkable deposits) plus savings deposits, small-denomination time deposits (those issued in amounts of less than $100,000), and retail money market mutual fund shares.

Index of Manufacturers’ Prices
Percentage of purchasing agents who report paying higher prices in the current month compared with the preceding month. A higher index indicates stronger demand for business inputs relative to their supply.

Consumer Credit Outstanding
Percent change in the amount of consumer debt outstanding during the month from the amount three months earlier. Consumer debt includes credit card loans, auto loans, and education loans and so on, but not home mortgages or home equity loans. Borrowing is a source of consumer purchasing power.

New Housing Permits
New private housing units authorized by building permits. This variable tends to lead construction expenditures.

Initial Claims for State Unemployment Insurance
Inverted for analysis. Measures the average number of persons who file first-time claims for unemployment compensation each week in a given month. A decline in general business activity leads to layoffs.

Average Workweek In Manufacturing
The total of paid labor-hours of manufacturing production workers divided by the number of such workers. Employers tend to adjust the workweek of their labor force before they adjust the size of their workforce.

We obtain the total monthly returns/growth rates (%) for each variable, and use them in estimation. Descriptive statistics of the variables is presented in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barclays U.S. Government/Credit Index</td>
<td>321</td>
<td>0.58</td>
<td>1.26</td>
<td>-4.19</td>
<td>4.53</td>
</tr>
<tr>
<td>Barclays U.S. Treasury Bond Index</td>
<td>321</td>
<td>0.55</td>
<td>1.32</td>
<td>-4.39</td>
<td>5.31</td>
</tr>
<tr>
<td>Barclays U.S. High-Yield Bond Index</td>
<td>321</td>
<td>0.74</td>
<td>2.54</td>
<td>-15.90</td>
<td>12.11</td>
</tr>
</tbody>
</table>
The descriptive statistics show that mean total monthly returns are higher in the stock and bond market, compared with the monthly growth rates in the real economy. Particularly, mean growth rates to three macroeconomic variables (new housing permits, initial claims for unemployment insurance, and average workweek in manufacturing) are the lowest among all variables.

3. Model and Methodology

The model implemented in this study is the single-factor and multi-factor version of the dynamic unobserved factor model used in Kose, Otrok, and Whiteman (2003).

Let T denote the length of time series. Observable growth rates of these variables are denoted $y_{i,t}$ for variable $i = 1,...,12$ and time period $t=1,...,T$.

The dynamic single-factor model decomposes dynamic of observables $y_{i,t}$ into the sum of two unobservable components:

- $f_{t}^{common}$ – the common factor, affects all variables
- $\epsilon_{i,t}$ - idiosyncratic factor, specific to each variable i

The single-factor model is:

$$y_{i,t} = a_{i} + b_{i}^{common} f_{t}^{common} + \epsilon_{i,t} \quad (1)$$
Where $a_i$ is a constant, $b_{i,\text{common}}$ is exposure or loading of series $i$ to the common factor.

Both components follow autoregressive processes of order 2:

$$f_{t,\text{common}} = \phi_{0,1} f_{t-1,\text{common}} + \phi_{0,2} f_{t-2,\text{common}} + u_{t,\text{common}}$$

(2)

$$\epsilon_{i,t} = \phi_{i,1} \epsilon_{i,t-1} + \phi_{i,2} \epsilon_{i,t-2} + \sigma_i u_{i,t}$$

(3)

Where $\sigma_i$ - standard deviation of idiosyncratic component,

$u_{i,t} \sim N(0,1)$ for $i=0$ - innovation to equation (2), and $i=1,\ldots,12$ - innovations to equation (3) (Kose, Otrok, and Whiteman 2003).

The dynamic multi-factor model decomposes dynamic of observables $y_{i,t}$ into the sum of several unobservable components:

$f_{t,\text{common}}$ - common factor, affects all variables.

$f_{t,\text{sector}}$ - sector-specific factor, affects a sub-group of variables. Two factors naturally come to mind here: one affects only the real economy, while the other one affects only the financial market.

$\epsilon_{i,t}$ - idiosyncratic factor, specific to each $i$.

The multi-factor model is:

$$y_{i,t} = a_i + b_{i,\text{common}} f_{t,\text{common}} + b_{i,\text{sector}} f_{t,\text{sector}} + \epsilon_{i,t}$$

(4)

Where $a_i$ is a constant, $b_{i,\text{common}}$ is exposure or loading of series $i$ to the common factor, $b_{i,\text{sector}}$ is exposure or loading of series $i$ to sector-specific factors.

All components follow autoregressive processes of order 2:

$$f_{t,\text{common}} = \phi_{0,1} f_{t-1,\text{common}} + \phi_{0,2} f_{t-2,\text{common}} + u_{t,\text{common}}$$

(5)

$$f_{t,\text{sector}} = \phi_{0,1} f_{t-1,\text{sector}} + \phi_{0,2} f_{t-2,\text{sector}} + u_{t,\text{sector}}$$

(6)

$$\epsilon_{i,t} = \phi_{i,1} \epsilon_{i,t-1} + \phi_{i,2} \epsilon_{i,t-2} + \sigma_i u_{i,t}$$

(7)

Where $\sigma_i$ - standard deviation of idiosyncratic component,

$u_{i,t} \sim N(0,1)$ for $i=0$ – innovations to equation (5) and (6) and $i=1,\ldots,n$ – innovations to equation (7).

In order to find how significant the common factors are in explaining the variation of the observable variables, we use variance decomposition (Kose, Otrok, and Whiteman 2003). We decompose the
variance of each observable variable \( y_{i,t} \) into the fraction that is due to common factors \( f_{t}^{\text{common}}, f_{t}^{\text{sector}} \), and the idiosyncratic component \( \epsilon_{i,t} \).

\[
\text{Var}(y_{i,t}) = (b_{i}^{\text{common}})^2 \text{Var}(f_{t}^{\text{common}}) + (b_{i}^{\text{sector}})^2 \text{Var}(f_{t}^{\text{sector}}) + \text{Var}(\epsilon_{i,t})
\]

For example, the fraction of volatility due to common factor would be:

\[
\frac{(b_{i}^{\text{common}})^2 \text{Var}(f_{t}^{\text{common}})}{\text{Var}(y_{i,t})}
\]

Because the factors are unobservable, special methods must be employed to estimate the model. Our empirical model uses the Markov Chain Monte Carlo algorithm and Gibbs sampling (see the Appendix for estimation details for each variable). Because the full set of conditional distributions is known, it is possible to generate random samples from the joint posterior distribution for the unknown parameters and the unobserved factor using a Markov-Chain Monte Carlo (MCMC) algorithm. Following Kose, Otrok, and Whiteman (2003), we use the Gibbs sampling procedure, which takes this complex problem and decomposes it into a set of tractable ones. We take initial values of the parameters and factors as given, and first sample from the posterior distribution of the parameters conditional on the factors; next we sample from the distribution of the common factor conditional on the parameters and the sector factors; then we sample each sector factor conditional on the common factor. We run 14,000 iterations (the first 4,000 iterations are discarded) to ensure the convergence of results.

4. Empirical Results

We first run the one-factor model, which assumes only one common factor that explains co-movement between economic conditions and financial market performance. No sectoral co-movements are taken into account. Chart 1 shows the three-month moving average of the common factor.

Chart 1: Co-movement captured in the one-factor model
The common factor shown in Chart 1 reveals several economic and financial downturns; for instance, the recession of 1990, the dot-com bubbles during the late 1990s and early 2000s, and the financial crisis of 2008. But Chart 1 doesn’t provide information on the significance of the so-called common factor in terms of explaining volatilities of financial markets and the real economy. Variance decomposition is performed to answer this question. See results in Table 3.

Table 3: Shares of variation in each variable explained by the common factor

<table>
<thead>
<tr>
<th>Variable</th>
<th>Common factor share</th>
<th>Idiosyncratic component share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barclays U.S. Government/Credit Bond Index</td>
<td>0.11%</td>
<td>99.89%</td>
</tr>
<tr>
<td>Barclays U.S. Treasury Bond Index</td>
<td>0.40%</td>
<td>99.60%</td>
</tr>
<tr>
<td>Barclays U.S. Corporate High Yield Index</td>
<td>18.13%</td>
<td>81.87%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>50.99%</td>
<td>49.01%</td>
</tr>
<tr>
<td>NASDAQ 100</td>
<td>38.16%</td>
<td>61.84%</td>
</tr>
<tr>
<td>Russell 2000</td>
<td>52.76%</td>
<td>47.24%</td>
</tr>
<tr>
<td>M2 money supply</td>
<td>0.07%</td>
<td>99.93%</td>
</tr>
<tr>
<td>Index of manufacturers' prices</td>
<td>0.20%</td>
<td>99.80%</td>
</tr>
<tr>
<td>Consumer credit outstanding</td>
<td>0.06%</td>
<td>99.94%</td>
</tr>
<tr>
<td>New housing permits</td>
<td>0.81%</td>
<td>99.19%</td>
</tr>
<tr>
<td>Initial claims for unemployment insurance</td>
<td>1.54%</td>
<td>98.46%</td>
</tr>
<tr>
<td>Average workweek in manufacturing</td>
<td>0.03%</td>
<td>99.97%</td>
</tr>
</tbody>
</table>

Note: 100%-common factor share = idiosyncratic component share
The results show that the stock market variables are dominant in the common factor. Fractions of variation in the stock market indexes that are explained by the common factor are substantially larger than those in any other variables. The corporate high-yield index seems to be more correlated with stocks than other bond indexes. The common factor can barely account for variation of any economic variables selected. In other words, evidence of co-movement between the real economy and financial markets is lacking.

One of the reasons that can explain the lack of co-movements is that the strong correlation between stock indexes takes over the common factor, leaving little room for the bond indexes and economic variables to participate. Therefore, we extend the model by introducing three sectoral factors. Each sector factor captures co-movement within a subgroup of variables. For example, the bond market sector factor accounts for the co-movement among bond indexes. There are four factors in the adjusted model: the bond market sector factor, the stock market sector factor, the real economy sector factor, and the common factor. The sector factors are designed to separate sectoral co-movement effects from the co-movement between financial markets and the economy. The results are shown in Chart 2 and Table 4.

Chart 2: Co-movements captured in the four-factor model
Table 4: Shares of variation in each variable explained by the common factor and the sector factor

<table>
<thead>
<tr>
<th></th>
<th>Common factor share</th>
<th>Sector factor share</th>
<th>Idiosyncratic component share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>sector factor—bond</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barclays U.S. Government/Credit</td>
<td>99.05%</td>
<td>0.08%</td>
<td>0.87%</td>
</tr>
<tr>
<td>Bond Index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barclays U.S. Treasury Bond</td>
<td>98.95%</td>
<td>0.09%</td>
<td>0.96%</td>
</tr>
<tr>
<td>Index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barclays U.S. Corporate High-Y</td>
<td>54.53%</td>
<td>0.12%</td>
<td>45.35%</td>
</tr>
<tr>
<td>Yield Index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>sector factor—stock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>30.30%</td>
<td>37.19%</td>
<td>32.51%</td>
</tr>
<tr>
<td>NASDAQ 100</td>
<td>9.27%</td>
<td>37.14%</td>
<td>53.59%</td>
</tr>
<tr>
<td>Russell 2000</td>
<td>1.22%</td>
<td>51.53%</td>
<td>47.25%</td>
</tr>
<tr>
<td><strong>sector factor—economy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2 money supply</td>
<td>78.81%</td>
<td>1.03%</td>
<td>20.16%</td>
</tr>
<tr>
<td>Index of</td>
<td>26.40%</td>
<td>10.10%</td>
<td>63.50%</td>
</tr>
</tbody>
</table>
On the economy side, money/credit-related indicators appear more correlated with financial markets than other economic variables, such as labor market indicators and the housing market indicator. The co-movement between the real economy and financial markets is able to explain 91 percent of variation in the monthly growth rates of consumer credit and 79 percent of the M2 money supply. Within the economy, there is co-movement (or business cycle) that didn’t spread out to financial markets. Such

<table>
<thead>
<tr>
<th></th>
<th>manufacturers' prices</th>
<th>Consumer credit outstanding</th>
<th>New housing permits</th>
<th>Initial claims for unemployment insurance</th>
<th>Average workweek in manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>91.25%</td>
<td>91.25%</td>
<td>8.70%</td>
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<td>0.05%</td>
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<td>66.40%</td>
<td>66.40%</td>
<td>66.40%</td>
</tr>
</tbody>
</table>

Note: 100%-common factor share-sector factor share=idiosyncratic component share

Chart 2 provides four co-movements captured in the four-factor model: co-movement (i.e., common factor) between economic growth and financial market returns; co-movement within the stock market or the stock market cycle (i.e., stock market sector factor); co-movement within the bond market or the bond market cycle (i.e., bond market sector factor); and the co-movement within the economy or the business cycle (i.e., economy sector factor). Table 4 shows the results of variance decomposition to demonstrate how important the co-movements are to explain variation of returns/growth rates for each variable.

As shown in Table 4, the dominant co-movement among stock indexes in the previous model is transformed to the stock market sector factor. The stock market sector factor is able to explain a big portion of variability for all three stock indexes returns. In other words, stock indexes seem to co-move with one another. But the S&P 500 shows a high correlation with other variables besides its correlation with two other stock indexes. This is to say that the S&P 500 is more correlated with the bond market and economic conditions than NASDAQ 100 and the Russell 2000.

Another important observation is that after separating the effect of sectoral co-movement in the stock market, bond indexes appear highly co-moved with economic variables. Almost 99 percent of variation of the U.S. government/credit bond index and the U.S. Treasury bond index can be accounted for by the co-movement between financial markets and the real economy. The U.S. corporate high-yield index seems less related to the other bond indexes; about 45 percent of variation in the U.S. corporate high-yield index returns is not accounted for by either the common factor or the bond market sector factor. The possible reason is that the U.S. corporate high-yield index is partially co-moved with stock indexes as the one-factor model shows earlier, and such co-movement is not designed to be captured by either the common factor or the bond market sector factor, and hence remains as an idiosyncratic component share.

On the economy side, money/credit-related indicators appear more correlated with financial markets than other economic variables, such as labor market indicators and the housing market indicator. The co-movement between the real economy and financial markets is able to explain 91 percent of variation in the monthly growth rates of consumer credit and 79 percent of the M2 money supply. Within the economy, there is co-movement (or business cycle) that didn’t spread out to financial markets. Such
sectoral co-co-movement is mainly contributed by the initial claims for unemployment insurance, new housing permits, and the index of manufacturers’ prices.

Nevertheless, the variable that indicates the average workweek in manufacturing is shown to be little correlated with both financial markets and other economic conditions. In future work, we will consider substituting this indicator with other variables.

5. Conclusions

This paper aims to investigate the presence of long-term co-movements between the real economy and financial markets by implementing dynamic factor models. The one-factor model is first carried out in which there is only one common factor designed to explain the co-movement between financial markets and the economy. The results show that the co-movement only explains the correlation among stock indexes but fails to account for bond indexes and economic variables. In other words, the one-factor model only provides evidence of stock market co-movement, not the co-movement between bond and stock markets and the economy.

Next, we extend the model by including sector factors that represent sectoral co-movements. The purpose of including sector factors is to separate sectoral co-movements from the cross-sector co-movement. For instance, the stock market sector factor explains the co-movement among stock indexes. Separating the sectoral co-movement allows the model to capture the potentially different strength of co-movement across sectors and within each sector. The main findings are threefold: first, bond indexes are more correlated with economic variables, compared with stock indexes. Second, stock indexes are co-moved with one another, but correlated little with bond indexes or with economic variables. Last but not least, among the economic indicators, money/credit-related indicators (i.e., the money supply and consumer credit) perform better than the others in terms of predicting the performance of financial markets.

The results have several important implications for long-term investors. For bond investors, bond returns, especially government bonds and high-grade corporate bonds, are proved to be highly co-moved with growth rates of the money supply and consumer credit in the past three decades. Looking at movement trends of those money/credit indicators provides a useful perspective of bond index performance. For stock investors, little evidence of long-term correlation between stocks and bonds or economic conditions is found. However, the S&P 500 seems to be correlated more with bonds and economic variable than the Nasdaq 100 and the Russell 2000. About half of the variation of the Nasdaq 100 and the Russell 2000 cannot be explained either by the common factor or by the stock market sector factor. This is to say that stock investment is risky, and the performance of the bond market and the economy can help little in accounting for stock return fluctuations. A substantial portion of stock return variations (about half) remains unexplained.
References

The Real Economy


Stock Prices’ Co-Movements


The Financial Market and the Real Economy


**Methodology**


Appendix: Estimating the dynamic factor model by using Gibbs-sampling


The baseline model can be rewritten in state space model pattern:

\[ y_{it} = a_i + [\lambda_{i1} 0 \lambda_{i2} 0 1 0] \]

Subject to:

\[
\begin{bmatrix}
    f^w_t \\
    f^w_{t-1} \\
    f^r_t \\
    f^r_{t-1} \\
    u_{it} \\
    u_{i,t-1}
\end{bmatrix} =
\begin{bmatrix}
    \phi_1^w & \phi_2^w & 0 & 0 & 0 \\
    1 & 0 & 0 & 0 & 0 \\
    0 & 0 & \phi_1^r & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & q_{i1} & q_{i2} \\
    0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    f^w_{t-1} \\
    f^w_{t-2} \\
    f^r_{t-2} \\
    f^r_{t-2} \\
    u_{i,t-1} \\
    u_{i,t-2}
\end{bmatrix} + \begin{bmatrix}
    \varepsilon^w_t \\
    \varepsilon^r_t \\
    0 \\
    0 \\
    e_{it} \\
    0
\end{bmatrix}
\]

Gibbs-sampling for estimating parameters is showed in this section as follows:

For generating \( \Psi \) for each country \( i \), we know that

\[ y_i = A + \Lambda f_i + u_i \]

\[ u_{it} = \psi_{i1} u_{i,t-1} + \psi_{i2} u_{i,t-2} + e_{it} \]

So, in matrix notation, we can get

\[ \Psi \Psi = U_i \Psi + \Psi \Psi, \quad \Psi \Psi : N(0, \sigma^2_k I_\gamma) \]

Prior distribution is assumed to be \( \Psi \Psi : N(a, b) \).
Posterior distribution can be calculated as

\[ \theta_i \mid \Lambda, \sigma_i^2, f_i, y_i : N(a^*_i, b^*_i) \]

where

\[ a^*_i = \left( b_i^{-1} + \sigma_i^{-2} U_i' U_i \right)^{-1} (b_i^{-1} a_i + \sigma_i^{-2} U_i' \theta_i) \]

\[ b^*_i = \left( b_i^{-1} + \sigma_i^{-2} U_i' U_i \right)^{-1} \]

For generating \( \Phi \), we have

\[ f_i = \Phi_1 f_{i-1} + \Phi_2 f_{i-2} + \epsilon_i \]

Prior distribution: \( \theta_0 \sim N(c_i, d_i) \)

Posterior distribution:

\[ \theta_i \mid f_i, y_i : N(c^*_i, d^*_i) \quad i = 1, 2, 3 \]

where

\[ c^*_i = \left( d_i^{-1} + F_i' F_i \right)^{-1} \left( d_i^{-1} c_i + F_i' f_i + \theta_i \right) \]

\[ d^*_i = \left( d_i^{-1} + F_i' F_i \right)^{-1} \]

For generating \( \sigma_i^2 \), we know from above

\[ \theta_i = U_i \theta_{i+1} + \theta_{i+2} \]

\( \theta_0 \sim N(0, \sigma_i^2 I_T) \)

Prior distribution is

\[ 1/\sigma_i^2 : \Gamma\left( \frac{v_i}{2}, \frac{w_i}{2} \right) \]

Posterior distribution is

\[ 1/\sigma_i^2 \mid \theta_i, \Lambda, f_i, y_i : \Gamma\left( \frac{v_i + (T - 2)}{2}, \frac{w_i + (\theta_i' - U_i \theta_{i+1})'(\theta_i' - U_i \theta_{i+1})}{2} \right) \]
For generating $\Lambda$, we need to do some adjustment. Substitute $u_t = \Psi_1 u_{t-1} + \Psi_2 u_{t-2} + e_t$ into $y_t = \Lambda + f_t + u_t$. Take $i=1$ for example,

$$
\begin{align*}
    y_{1t} &= \lambda_{11} f_t^w + \lambda_{12} f_t^d + u_{1t} \\
    u_{1t} &= \psi_{11} u_{t-1} + \psi_{12} u_{t-2} + e_{1t}
\end{align*}
$$

Then, we can get

$$
\begin{align*}
    y_{1t} - \lambda_{11} f_t^w + \lambda_{12} f_t^d &= \psi_{11} (y_{1t-1} - \lambda_{11} f_{t-1}^w + \lambda_{12} f_{t-1}^d) + \psi_{12} (y_{1t-2} - \lambda_{11} f_{t-2}^w + \lambda_{12} f_{t-2}^d) + e_{1t} \\
    y_{1t} - \lambda_{11} y_{1t-1} - \lambda_{12} y_{1t-2} &= \lambda_{11} (f_t^w - \psi_{11} f_{t-1}^w - \psi_{12} f_{t-1}^d) + \lambda_{12} (f_t^d - \psi_{11} f_{t-1}^d - \psi_{12} f_{t-1}^d) + e_{1t} \\
    y_{1t}^* &= \lambda_{11} f_t^w + \lambda_{12} f_t^d + e_{1t}
\end{align*}
$$

By using the same method of generating $\Phi$, we can get the sampling for $\Lambda$.

For estimating unobserved factors, we rewrote the model into a state space pattern and Kalman Filter is applied to achieve the estimate of factors.

It’s important to monitor the convergence of the computation. We did so in a number of ways. First, we restart the computation from a number of different initial values, and the procedure always converges to the same results. Second, we discard the first 4,000 drawings and take the next 10,000 drawings. We try more drawings and the results show the same.